Dr. Rahul Chauhan

UNIT - 3

Wave Optics

Interference and Diffraction

[Engineering Physics - I]

Unit-3

"Wave optics"

Interference

Interference - When the two light waves of the same frequency and having a constant phase difference traverse simultaneously in the same region of a medium and cross each other, then there is a modification in the intensity of light in the region of superposition, which is in general, different from the sum of intensities due to individual waves at that point. This modification in the intensity of light resulting from the superposition of two (or more) waves of light is called interference.

At certain points, the waves superimpose in such a way that the resultant intensity is greater than the sum of the intensities due to individual waves. The interference produced at these points is called constructive interference or reinforcement, while at certain other points, the resultant intensity is less than the sum of the intensity due to individual waves. The interference produced at these points is called destructive interference.

Coherent sources → Two sources of light are said to be coherent if they emit light which have always a constant phase difference between them.

Types of interference (Formation of co-herent sources in practice)

The phenomenon of interference may be grouped into two categories depending upon the formation of two coherent sources in practice.

1 - **Division of amplitude** — In this method, the amplitude of the incident beam is divided into two or more parts either by the partial reflection or refraction. Thus we have coherent beams produced by division of amplitude.

2 - **Division of wave front** — under this category, the coherent sources are obtained by dividing the wave front, originating from a common source. This class of interference requires essentially a point source or a narrow slit source.

Wave front

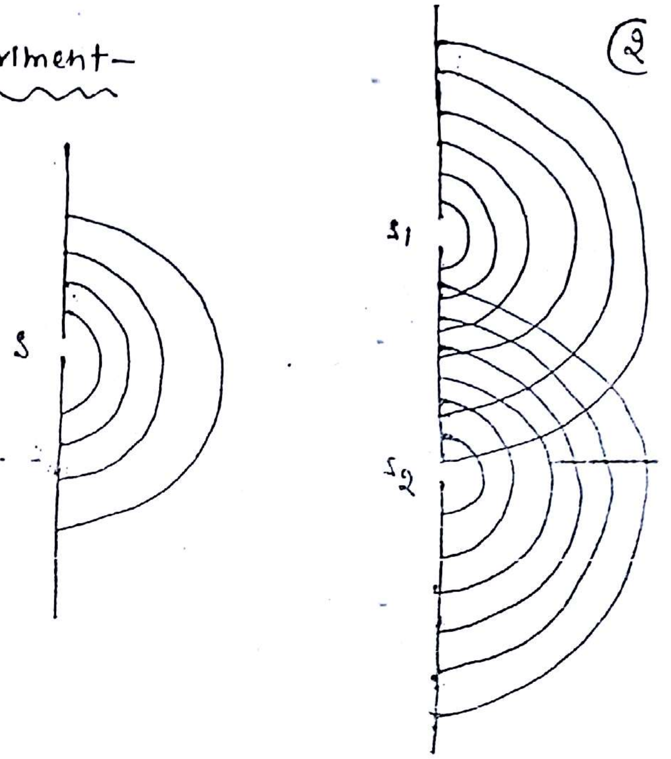
A wave front is defined as the locus of such points which are in the same state of vibration i.e. in the same phase.

Methods of obtaining co-herent sources — In actual practice, it is not possible to have two independent coherent sources of light but for experimental purposes, two virtual sources of light are derived from a single source by some coherent devices. Any phase change in one is simultaneously accompanied by the same phase change in the other so that the phase difference between the two sources remain constant.

In some experimental setup, the coherent sources are obtained in the following manner:

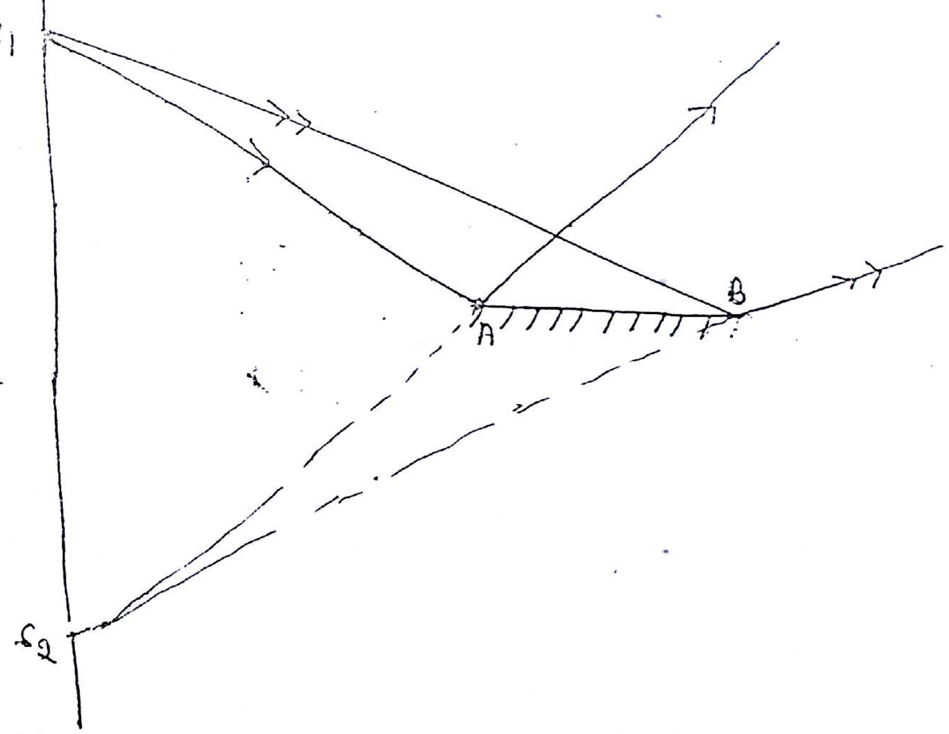
1 - **Young's Double - slit experiment-**

In this method monochromatic light is passed from a small source through a slit S and the light emerging from this slit is used to illuminate two other narrow slits S_1 and S_2 which are very closed together and parallel to S. S_1 and S_2 acts as a coherent sources, both being derived from S.



2- **Lloyd's mirror-**

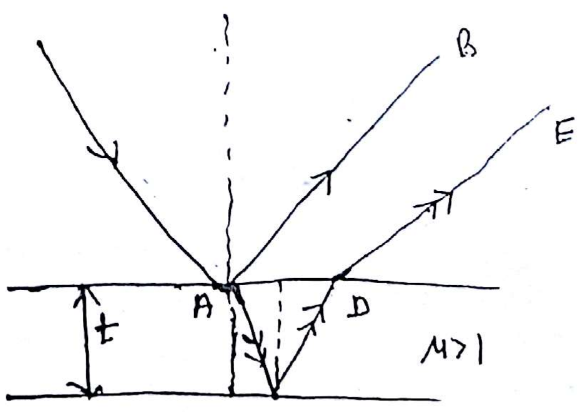
It is an arrangement to obtain two coherent sources of light to produce interference. A plane glass plate (AB, acting as mirror) is illuminated at almost grazing incidence by a monochromatic light from a narrow slit S_1. A virtual image S_2 of S_1 is formed closed to S_1 by reflection from the mirror. The real source S_1 and virtual source S_2 acts as coherent sources for the study of interference.



3- **Thin film (Reflected system)-**

In this case, the two coherent beams are obtained by division of the amplitude of a incoming beam by partial reflection and refraction.

Rays AB and DE are derived from the same incident wave SA and they are coherent and acts



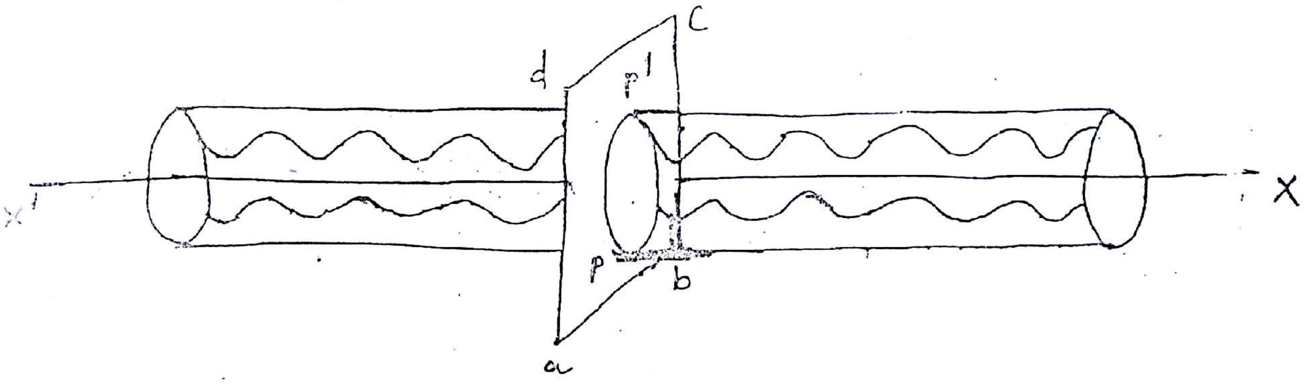
Types of coherence -

1- Spatial coherence

2- Temporal coherence

- Spatial coherence - A wave is said to posses spatial coherence if the phase difference of the waves crossing the two points lying on a plane perpendicular to the direction of propagation of the beam is time independent. This coherence is also termed as transverse or lateral coherence. It is a measure of the minimum seperation across the wave from where two waves remains constant.

Figure shows a beam of light travelling along the line X'X. Here abcd is a plane perpendicular to X'X. The beam is said to posses spatial coherence if the phase difference of the waves crossing P and P' at any instant is always constant.



- Temporal coherence - If the phase differences of waves crossing the two points lying along the direction of propagation of the beam is time dependent, then a beam of light is said to posses temporal or time coherence. This coherence is also known as longitudinal coherence.

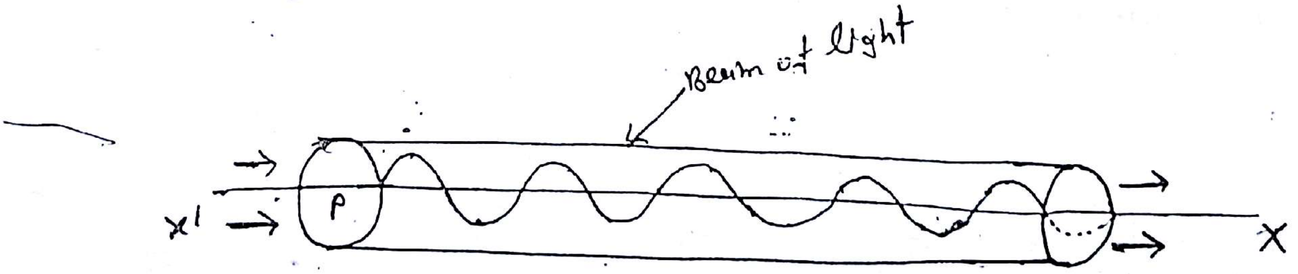
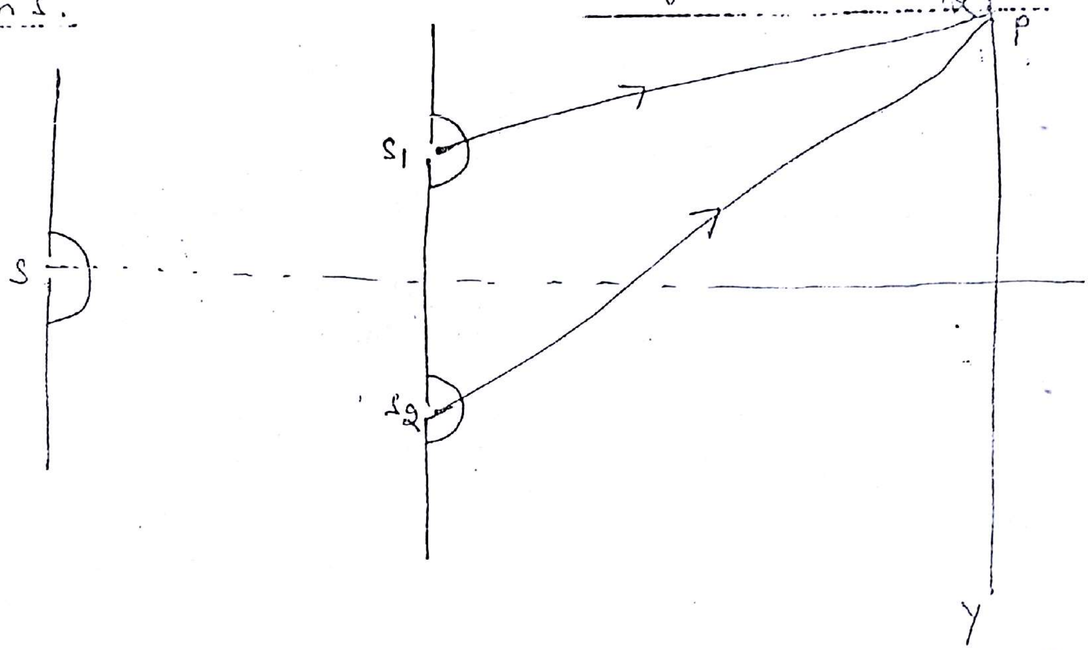


Fig. 3.8 shows a beam of light travelling along the axis X'X. P and Q are the two points lying on this line. The beam is said to posses temporal or time coherece if the phase difference of the waves crossing P and Q at any instant is always constant

Resultant Intensity due to superposition of two interfering waves –

Let us consider a monochromatic source of light s emitting waves of wavelength λ and let s_1 and s_2 are two similar parallel slits very close together and equidistant from s.



Let a_1 and a_2 be the amplitudes at P due to the waves from s_1 and s_2 respectively. The waves reached at point P after traversing different path s_1P and s_2P. Let the phase difference between two waves is δ.

$$\delta = \frac{2\pi}{\lambda}(s_2P - s_1P)$$

Let y_1 and y_2 are the displacements of two waves, then

$$y_1 = a_1 \sin\omega t$$
$$y_2 = a_2 \sin(\omega t + \delta)$$

By principle of superposition,

$$y = y_1 + y_2 = a_1 \sin\omega t + a_2 \sin(\omega t + \delta)$$
$$= a_1 \sin\omega t + a_2 \sin\omega t \cos\delta + a_2 \cos\omega t \sin\delta$$
$$= \sin\omega t (a_1 + a_2\cos\delta) + \cos\omega t (a_2 \sin\delta)$$

Let $a_1 + a_2\cos\delta = R\cos\theta$ — (1), $a_2 \sin\delta = R\sin\theta$ — (2)

(R and θ are new constants)

$y = \sin\omega t \, R\cos\theta + \cos\omega t \, R\sin\theta$

$y = R \sin(\omega t + \theta)$

Hence, the resultant displacement at P is simple harmonic and of amplitude R. Squaring and adding equation ① and ②, we get

$R^2\cos^2\theta + R^2\sin^2\theta = (a_1 + a_2\cos\delta)^2 + (a_2\sin\delta)^2$

$$\boxed{R^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta} \quad \text{—— ③}$$

Intensity (I) at P is proportional to the square of the resultant amplitude.

$$\boxed{I = R^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta} \quad \text{—— ④}$$

Hence resultant intensity is not just the sum of the intensities due to the separate waves $(a_1^2 + a_2^2)$.

For constructive interference or maximum intensity – According to equation ④, the intensity is depend on $\cos\delta$.

For maximum intensity, $\cos\delta = 1$

$\delta = 0, \pm 2\pi, \pm 4\pi, \ldots$

$\boxed{\delta = \pm 2n\pi}$ where $n = 0, 1, 2$

Path difference $(x) = \frac{\lambda}{2\pi} \cdot \delta = \frac{\lambda}{\cancel{2\pi}}(\pm \cancel{2}n\cancel{\pi})$

$\boxed{x = \pm n\lambda}$

From equation ④,

$I_{max} = a_1^2 + a_2^2 + 2a_1a_2$

$\boxed{I_{max} = (a_1 + a_2)^2}$

For destructive interference or minimum intensity–

$\cos\delta = -1$

$\delta = \pm\pi, \pm 3\pi, \pm 5\pi, \ldots$

$\boxed{\delta = \pm(2n+1)\pi}$, where $n = 0, 1, 2, 3 \ldots$

or $\pm(2n-1)\pi$, where $n = 1, 2, 3, \ldots$

Now, $x = \frac{\lambda}{2\pi} \cdot \delta = \frac{\lambda}{2\cancel{\pi}}(2n+1)\cancel{\pi}$

$$\boxed{\begin{aligned} x &= \pm(2n+1)\frac{\lambda}{2}, \text{ where } n = 0, 1, 2, 3, \ldots \\ \text{or } x &= \pm(2n-1)\frac{\lambda}{2}, \text{ where } n = 1, 2, 3, \ldots \end{aligned}}$$

From equation 4;

$$I_{min} = a_1^2 + a_2^2 - 2a_1a_2$$

$$\boxed{I_{min} = (a_1 - a_2)^2}$$

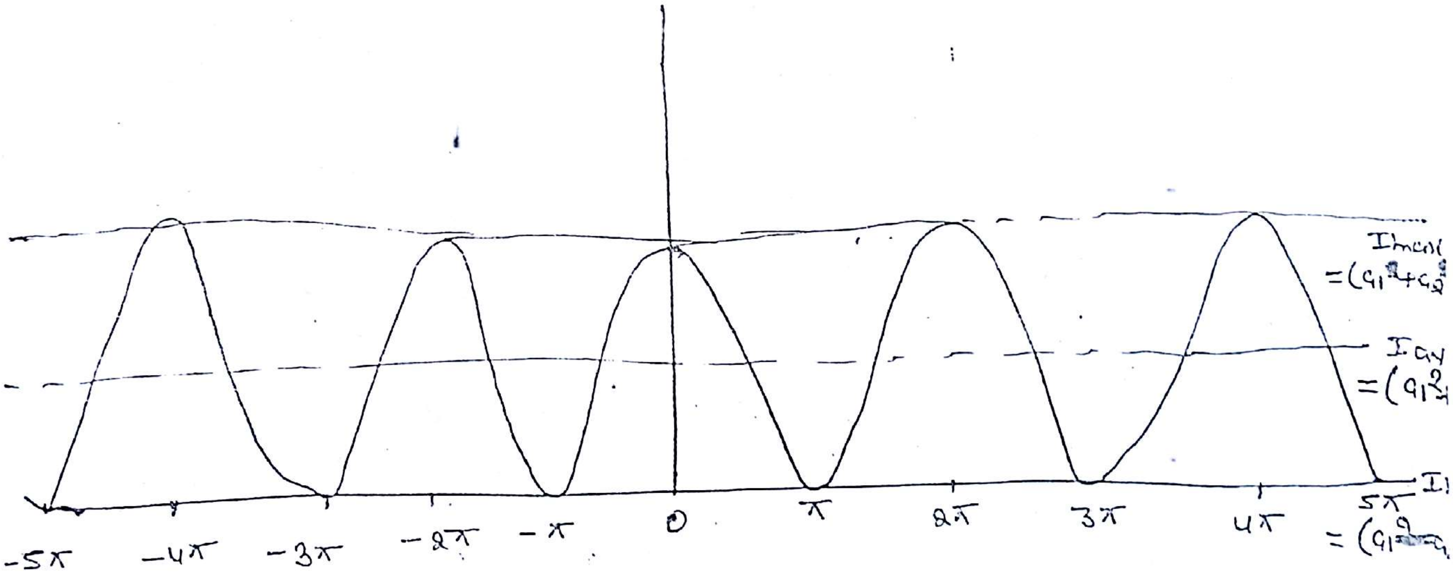
Average intensity —

$$I_{av} = \frac{I_{max} + I_{min}}{2} = \frac{(a_1+a_2)^2 + (a_1-a_2)^2}{2}$$

$$= \frac{2a_1^2 + 2a_2^2}{2}$$

$$\boxed{I_{av} = a_1^2 + a_2^2}$$

Intensity distribution curve —



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Interference in thin film by reflected light–

We need broad source for the interference in thin films.

Consider a parallel-side transparent thin film of thickness t and refractive index $\mu\ (>1)$. A ray of monochromatic light SA incident at an angle i, is partly reflected along AB and partly refracted along AC at angle r. At C, it is again partly reflected along CD as shown in figure. Similar reflection and refraction occurs at D. Light ray AB and DE produced interference in the eye.

Note point

Stokes Law– ① When a light ray reflected from the rarer to denser medium surface, then a phase of π angle in reflected wave is produced in comparison to incident wave.

② If reflection got from the denser medium or reflection got from that surface beyond which a rarer medium is present, then there is no change in phase.

③ The part of amplitude which reflected is given by the:

$$\boxed{r = \frac{\mu-1}{\mu+1}}$$

④ The part of amplitude which transmitted by refraction is,

$$\boxed{r^2 + t^2 = 1}$$

Path difference = Path ACD in medium − Path AL in air

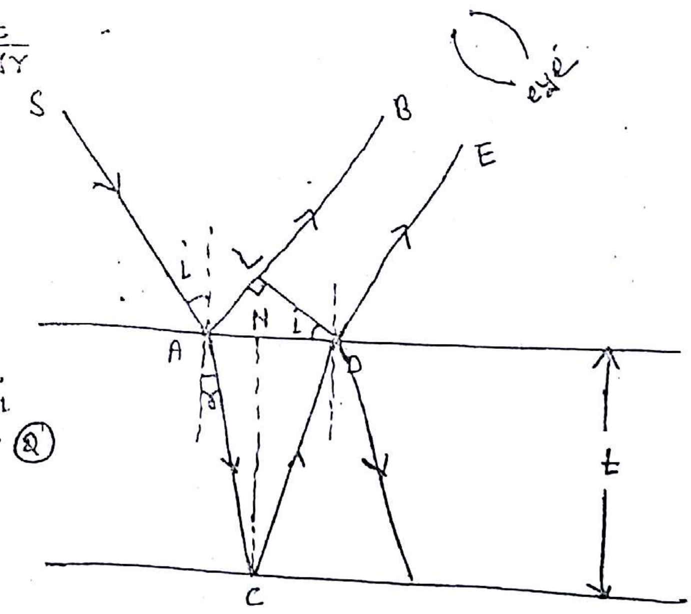
$$x = \mu(AC+CD) - AL \quad \text{——— (1)}$$

In ΔACN,

$$\cos r = \frac{CN}{AC} = \frac{t}{AC} \Rightarrow AC = \frac{t}{\cos r}$$

Similarly,

$$CD = \frac{t}{\cos r}$$



In ΔADL,

$$\sin i = \frac{AL}{AD}$$

$$AL = AD\sin i = (AN+ND)\sin i \quad \text{——— (2)}$$

In ΔACN,

$$\tan r = \frac{AN}{CN}$$

$$\frac{AN}{t} = \tan r$$

$$AN = t\tan r$$

Similarly, $ND = t\tan r$

From equation (2),

$$AL = (t\tan r + t\tan r)\sin i = (2t\tan r)\sin i = 2t\tan r \cdot \mu\sin r \left(\because \mu = \frac{\sin i}{\sin r}\right)$$

$$AL = \frac{2\mu t\sin^2 r}{\cos r}$$

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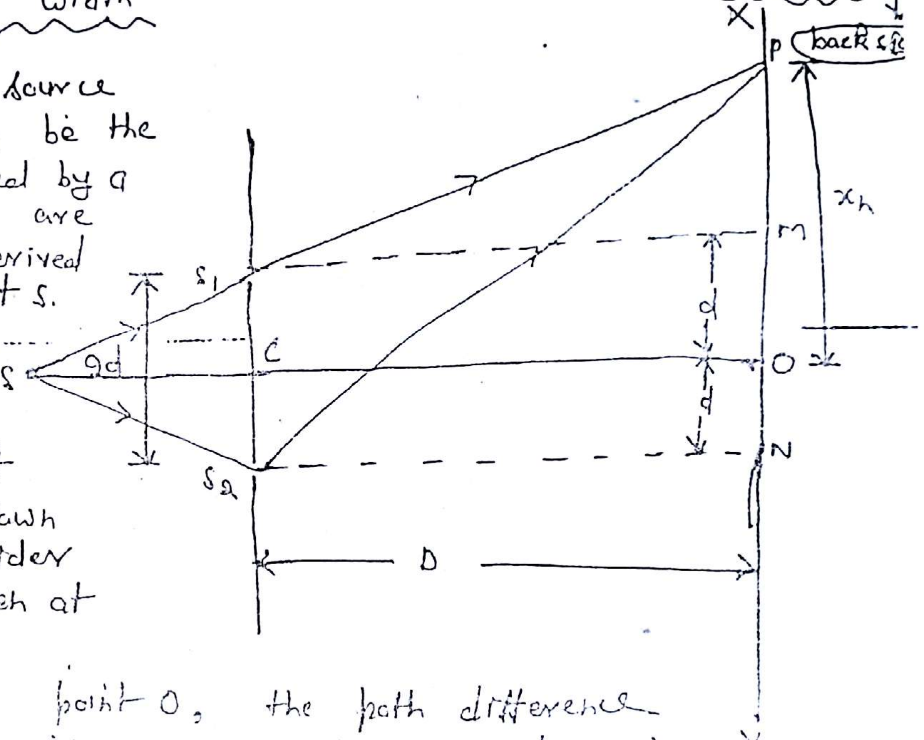
Expression of Fringe width—

Let S be a narrow source of light and S_1 and S_2 be the two narrow slits separated by a distance 2d. S_1 and S_2 are thus coherent sources derived from a source of light S. XY be a screen placed at a distance D from the coherent sources and O be the foot of the perpendicular drawn from C. Let us consider a point P on the screen at a distance x from O.



At the point O, the path difference between the waves which emerge from S_1 and S_2, is zero. At this point, we get ~~zero~~ order bright fringe. On the both sides of O, we get bright and dark fringes of diff orders. Let at point P, we got the bright fringe of n order, then

$$S_2P - S_1P = n\lambda \quad \text{——— (1)}$$

if the dark fringe of nth order, then

$$S_2P - S_1P = (2n+1)\frac{\lambda}{2} \quad \text{——— (2)}$$

In ΔPS_1M,

$$(S_1P)^2 = (S_1M)^2 + (PM)^2$$

$$(S_1P)^2 = D^2 + (x_n - d)^2 = D^2\left[1 + \frac{(x_n-d)^2}{D^2}\right]$$

$$S_1P = D\left[1 + \frac{(x_n-d)^2}{D^2}\right]^{1/2}$$

$$S_1P = D\left[1 + \frac{(x_n-d)^2}{2D^2}\right]$$

$$S_1P = D + \frac{(x_n-d)^2}{2D}$$

In ΔPS_2N,

$$(S_2P)^2 = (S_2N)^2 + (PN)^2 = D^2 + (x_n+d)^2$$

similarly,

$$S_2P = D + \frac{(x_n+d)^2}{2D}$$

$$\text{Path difference} = S_2P - S_1P = D + \frac{(x_n+d)^2}{2D} - D - \frac{(x_n-d)^2}{2D}$$

$$= \frac{\cancel{x_n^2} + \cancel{d^2} + 2x_nd - \cancel{x_n^2} - \cancel{d^2} + 2x_nd}{2D}$$

$$S_2P - S_1P = \frac{2x_nd}{} \quad \text{——— (3)}$$

From equation ① and ③,

$$\frac{2x_n d}{D} = n\lambda$$

$$\boxed{x_n = \frac{Dn\lambda}{2d}}$$ ⇒ This shows the (distance of) nth order bright fringe from center.

Putting $n = 0, 1, 2, 3 \ldots$

$x_0 = 0$, $x_1 = \frac{D\lambda}{2d}$, $x_2 = \frac{2D\lambda}{2d}$, $x_3 = \frac{3D\lambda}{2d}$ .....

The distance between any two consecutive bright or dark fringes is called the fringe width (ω).

$$\omega = x_1 - 0 = x_2 - x_1 = x_3 - x_2 = \ldots$$

$$\boxed{\omega = \frac{D\lambda}{2d}}$$

From equation ② and ③,

$$\frac{2x_n d}{D} = (2n+1)\frac{\lambda}{2}$$

$$\boxed{x_n = \frac{(2n+1)D\lambda}{4d}}$$ ⇒ this shows the distance of nth order dark fringe from center.

Putting $n = 0, 1, 2 \ldots$

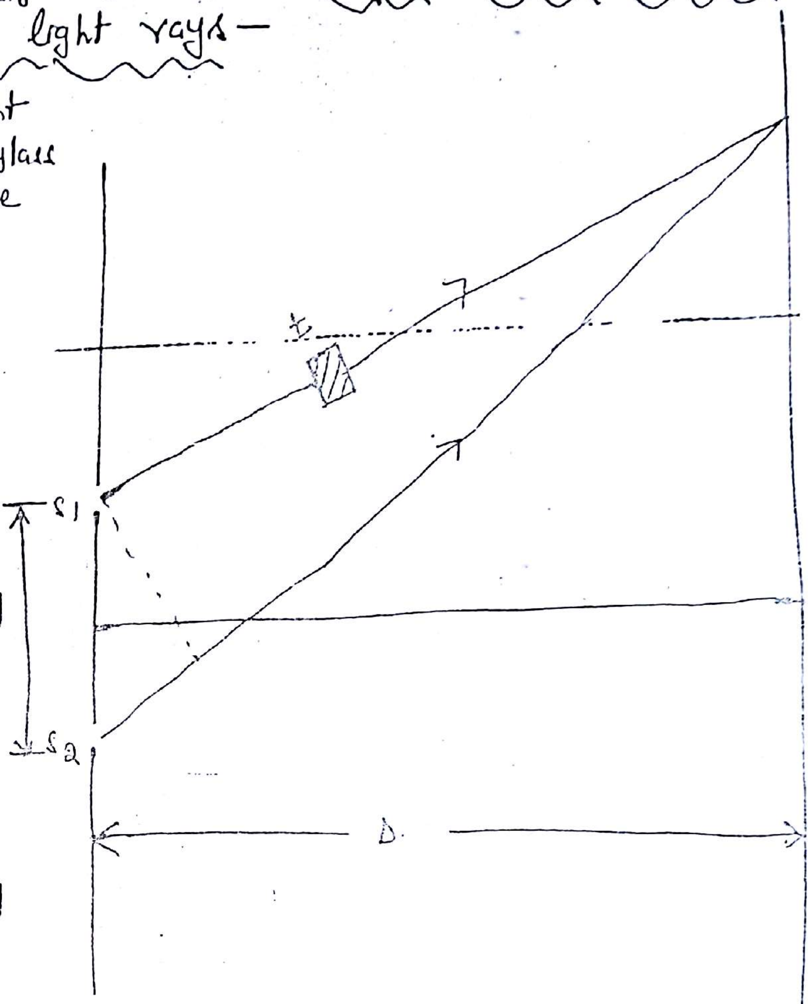
$x_0 = \frac{D\lambda}{4d}$, $x_1 = \frac{3D\lambda}{4d}$, $x_2 = \frac{5D\lambda}{4d}$, ...

$$\omega = (x_1 - x_0) = x_2 - x_1 = x_3 - x_2 = \ldots$$

$$\boxed{\omega = \frac{D\lambda}{2d}}$$

Thus the fringe width for bright or dark fringes is the same and is equal to $\frac{D\lambda}{2d}$. It is also independent from n.

Displacement of fringes on introducing a glass or mica plate in the path of light rays —



When a thin transparent plate of any material like glass or mica is introduced in the path of anyone of the interfering waves, there is a shift in the position of interference fringes.

Let S_1 and S_2 be two coherent monochromatic sources of wavelength λ and a thin plate of thickness t and refractive index μ is introduced in the path of the light from S_1.

The light from S_1 travels the path partly in air and partly in the plate. The distance travelled from S_1 in air is $(S_1P - t)$ and that in the plate of refractive index μ is t. If c and v be the velocities of light in air and in the plate respectively, then the time taken by light from S_1 to P is

$$= \text{Time in air} + \text{Time in medium}$$

$$= \frac{S_1P - t}{c} + \frac{t}{v}$$

$$\because v = \frac{c}{\mu} \text{ or } \mu = \frac{c}{v}$$

$$= \frac{S_1P - t}{c} + \frac{\mu t}{c} = \frac{S_1P + (\mu - 1)t}{c} \quad \text{——— (1)}$$

It is clear that the path from S_1 to P has increased by $(\mu-1)t$ as a result of introduction of the plat

The path difference between the waves which reached at point P $= S_2P - S_1P - (\mu-1)t$

$$= \frac{2xnd}{D} - (\mu-1)t \qquad \because S_2P - S_1P = \frac{2xnd}{D}$$

For bright fringe,

$$\text{Path difference} = n\lambda \Rightarrow \frac{2xnd}{D} - (\mu-1)t = n\lambda$$

$$\Rightarrow \boxed{x_n = \frac{D}{2d}\left[n\lambda + (\mu-1)t\right]} \quad \text{——— (2)}$$

if there is no plate, then $t=0$

$$\boxed{x_n' = \frac{Dn\lambda}{2d}}$$

The displacement of nth bright fringe is,

$$x_0 = x_n - x_n'$$

$$= \frac{D}{2d}\left[n\lambda + (\mu-1)t\right] - \frac{Dn\lambda}{2d}$$

$$= \frac{D}{2d}\left[\cancel{n\lambda} + (\mu-1)t - \cancel{n\lambda}\right]$$

$$\boxed{S = \frac{D}{2d}(\mu-1)t}$$

Since x_0 is independent from n, therefore all bright bright fringe are displaced through the same distance. The same displacement may also be obtained for dark fringes.

<u>Width of bright fringes —</u>

From equation (2),

$$x_n = \frac{D}{2d}\left[n\lambda + (\mu-1)t\right]$$

Putting $n = 0, 1, 2, 3, \ldots$

$$x_0 = \frac{D}{2d}(\mu-1)t, \quad x_1 = \frac{D}{2d}\left[\lambda + (\mu-1)t\right], \quad x_2 = \frac{D}{2d}\left[2\lambda + (\mu-1)t\right]$$

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Fringe width $(w) = x_1 - x_0 = x_2 - x_1 = \ldots$

$$\boxed{w = \frac{D\lambda}{2d}}$$

Hence the fringe width is not changed by introducing the plate.

Shape of interference fringes —

At point P, for maximum intensity,

$$S_2P - S_1P = n\lambda \quad \text{——— (1)}$$

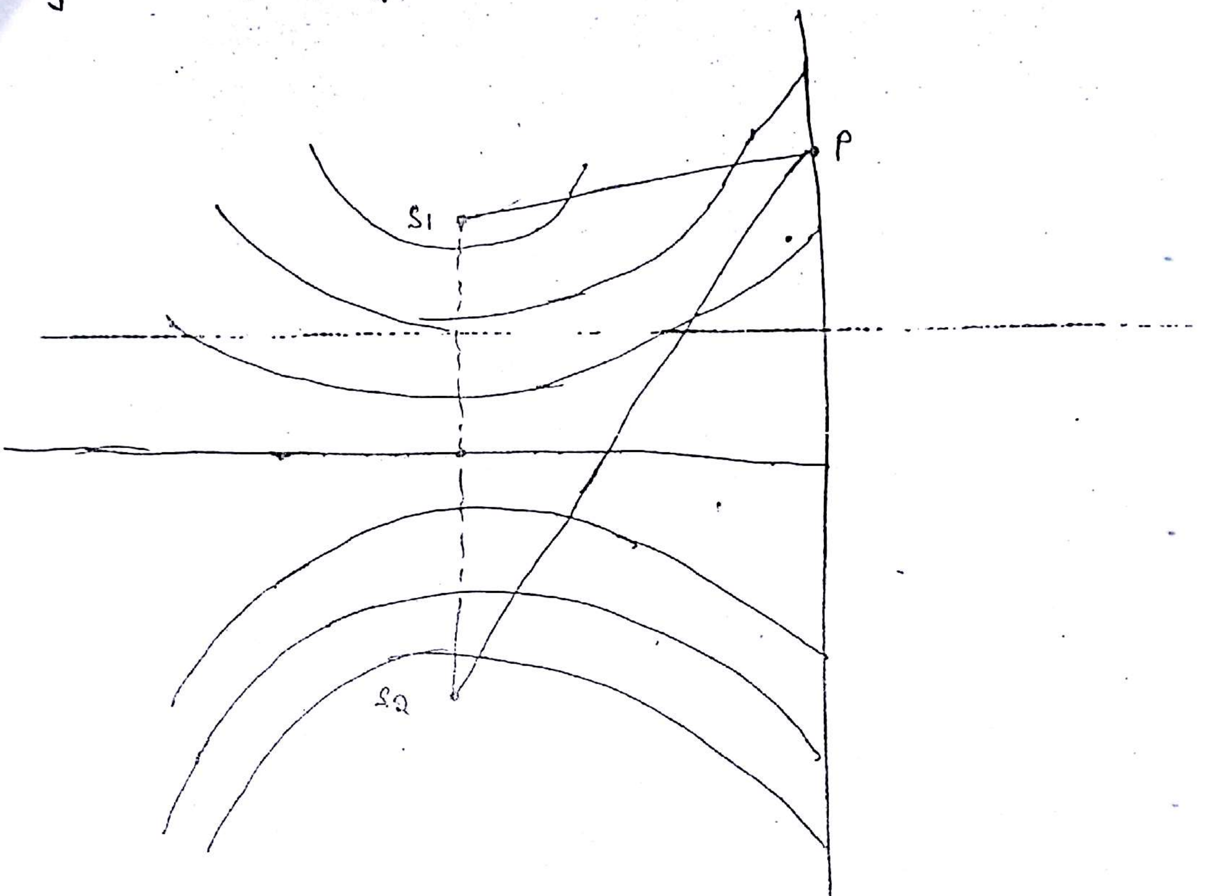
For minimum intensity,

$$S_2P - S_1P = (2n+1)\frac{\lambda}{2} \quad \text{——— (2)}$$

For the particular bright and dark fringe, n is constant. so that

$$\boxed{S_2P - S_1P = \text{constant}}$$

This is a equation of hyper-parabola which has two focus S_1 and S_2.



Conditions for sustained interference — To obtain a well defined and observable interference, the conditions for interference of light are as under:

① - The beams from two sources must propagate along the same line, otherwise the vibrations cannot be along a common lin

② - Two sources must be very narrow because a broad sour is equivalent to a number of soures and so positions of darkness and brightness cannot be well defined.

③ - The two sources must have equal intensities.

④ - The two soures should have equal amplitudes and frequ

⑤ - The common original source must be monochromatic i.e. emitting light of single wavelength.

⑥ - The light from the two sources must have either ze phase or constant difference.

From equation (1),

$$x = \mu\left(\frac{t}{\cos r} + \frac{t}{\cos r}\right) - \frac{2\mu t \sin^2 r}{\cos r} = \frac{2\mu t}{\cos r} - \frac{2\mu t \sin^2 r}{\cos r}$$

$$= \frac{2\mu t}{\cos r}\left(1 - \sin^2 r\right) = \frac{2\mu t}{\cancel{\cos r}} \cdot \cos^{\cancel{2}} r = 2\mu t \cos r$$

$$\boxed{x = 2\mu t \cos r}$$

$\because$ phase difference $= \frac{2\pi}{\lambda} \times x = \frac{2\pi}{\lambda} \times (2\mu t \cos r)$

$\therefore$ Resultant phase difference $(\delta) = \frac{2\pi}{\lambda}(2\mu t \cos r) - \pi$

For maximum intensity—

$$\delta = 2n\pi$$

$$\frac{2\pi}{\lambda}(2\mu t \cos r) - \pi = 2n\pi$$

$$\boxed{2\mu t \cos r = (2n+1)\frac{\lambda}{2}} \quad n = 0, 1, 2, \dots$$

or

$$\boxed{2\mu t \cos r = (2n-1)\frac{\lambda}{2}} \quad n = 1, 2, 3, \dots$$

For minimum intensity—

$$\delta = (2n-1)\pi$$

$$\frac{2\pi}{\lambda}(2\mu t \cos r) - \pi = (2n-1)\pi$$

$$\boxed{2\mu t \cos r = n\lambda} \quad n = 0, 1, 2, 3 \dots$$

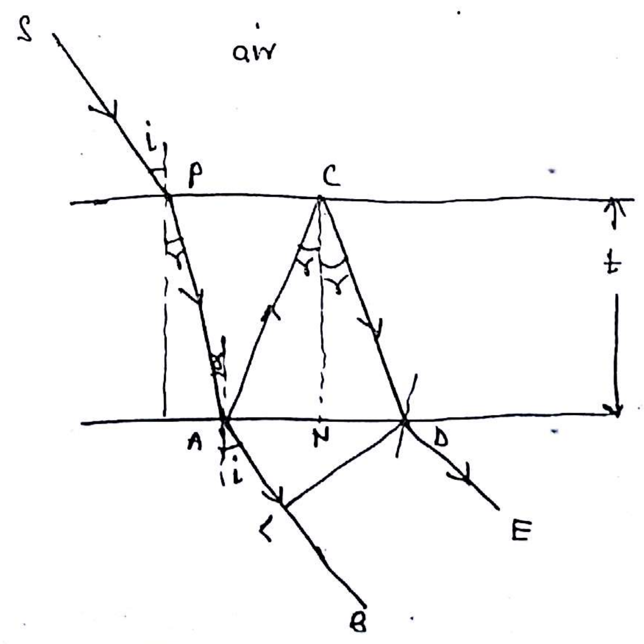
Interference in thin film by transmitted light—

Path difference (x) = path ACD in medium − path AL in air —(1)

From last article,

$x = 2\mu t \cos r$

$$(\delta) = \frac{2\pi}{\lambda}(2\mu t \cos r)$$



For maximum intensity,

$$\delta = 2n\pi$$

$$\frac{2\pi}{\lambda}(2\mu t\cos r) = 2n\pi$$

$$\boxed{2\mu t\cos r = n\lambda} \qquad n = 0, 1, 2, 3 \ldots$$

For minimum intensity—

$$\delta = (2n-1)\pi$$

$$\frac{2\pi}{\lambda}(2\mu t\cos r) = (2n-1)\pi$$

$$\boxed{2\mu t\cos r = (2n-1)\frac{\lambda}{2}} \qquad n = 1, 2, 3, \ldots$$

Colors in thin films — When a thin film is illuminated by monochromatic light and seen in reflected light, it will appear bright if $2\mu t\cos r = (2n-1)\frac{\lambda}{2}$ and dark if $2\mu t\cos r = n\lambda$. The path difference depends upon t (thickness of film), r and the inclination of the incident rays.

→ White light consists of a continuous range of wavelengths (i.e. colors). For ~~a particular value of t and r (i.e.~~ at a particular point of the film and for a particular position of eye), the waves of only certain wavelengths (colours) satisfy the condition of maxima. Therefore only those colors will be present in the reflected light with maximum intensity. The other neighbour wavelengths will be present with less intensity. There will also be certain wavelengths which satisfy the condition of minima. Such waves or colours will be absent from the reflected light. As a result the point of the film will appear coloured. Clearly, the colouration of the film varies with t and r, therefore if we vary either t and r, a different set of colours will be observed.

→ If the film is of uniform thickness everywhere and white light falls on it as a parallel beam, the path difference at each point of the film will be same and hence the entire film will appear uniformly coloured.

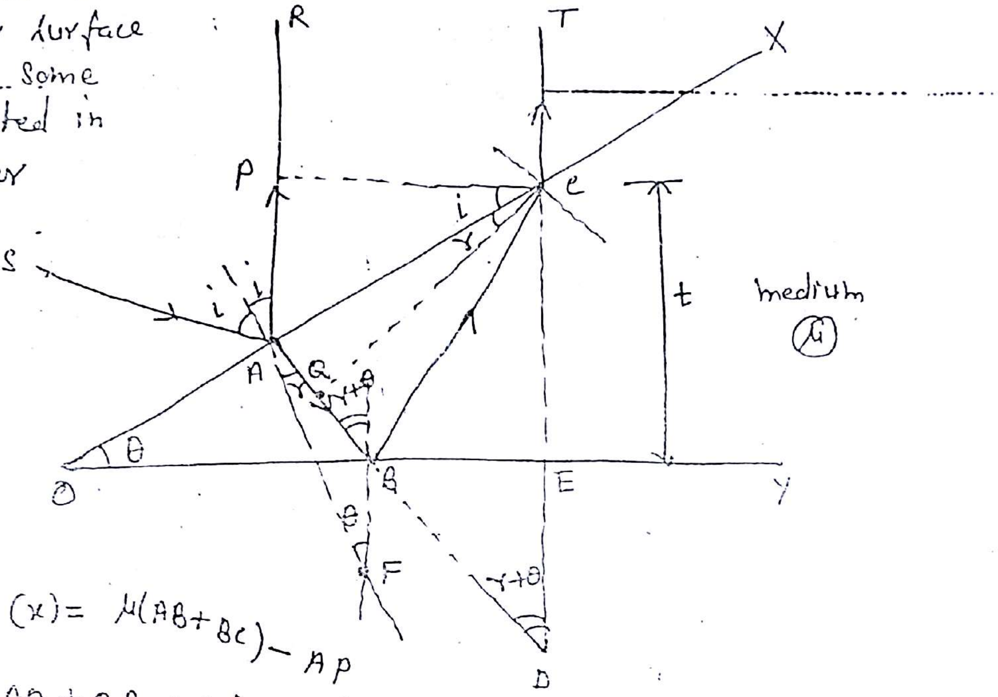
→ As the conditions of maxima and minima for transmit. light are opposite to those for reflected light, therefore, the colours visible in the reflected light will be complementary to the colours visible in transmitted light.

→ Colours are not observed in the case of thick film.

→ An excessively thin film appears black in reflected light.

Interference in wedge shaped thin film – A wedge-shaped film is one whose surfaces are inclined at a certain small angle. Figure shown a thin wedge-shaped film of refractive index μ, bounded by two ~~plated~~ surfaces OX and OY. Point O is called the 'wedge' of film and θ is the wedge angle.

The light ray SA is incident on the upper surface with incident angle i. Some part of it is reflected in AR direction and other part refracted in AB direction. At point B, it is again reflected in BC direction and transmitted in CT direction.



Path difference $(x) = \mu(AB + BC) - AP$

$x = \mu(AQ + QB + BC) - AP$ ——— (1)

From Snell's law –

$$\mu = \frac{\sin i}{\sin r} = \frac{AP/AC}{AQ/AC} = \frac{AP}{AQ} \Rightarrow AP = \mu AQ$$

Put in (1),

$x = \mu(QB + BC)$ ——— (2)

In ΔBCE and ΔBDE,

BE is common one

$\angle BEC = \angle BED = 90^0$

$\angle BCE = \angle BDE$

Hence the both triangle are equal.

$\Rightarrow BC = BD$, $CE = ED = t$ (let)

In ΔQDC,

$$\cos(r+\theta) = \frac{QD}{CD} = \frac{QD}{CE+ED} = \frac{QD}{2t} \Rightarrow QD = 2t\cos(r+\theta)$$

From equation (2),

$x = \mu(QB + BD) = \mu \cdot QD$

$$\boxed{x = 2\mu t\cos(r+\theta)}$$

$\therefore\ \delta = \frac{2\pi}{\lambda}\left[2\mu t\cos(r+\theta)\right]$

Resultant $\delta = \frac{2\pi}{\lambda}\left[2\mu t\cos(r+\theta)\right] - \pi$

For max. intensity—

$$\delta = 2n\pi \quad , \quad n = 0, 1, 2 \cdots$$

$$\frac{2\pi}{\lambda}\left[2\mu t \cos(r+\theta)\right] - \pi = 2n\pi$$

$$\boxed{2\mu t \cos(r+\theta) = (2n+1)\frac{\lambda}{2}} \qquad n = 0, 1, 2, \cdots$$

or $$\boxed{2\mu t \cos(r+\theta) = (2n-1)\frac{\lambda}{2}} \qquad n = 1, 2, 3 \cdots$$

For min. intensity—

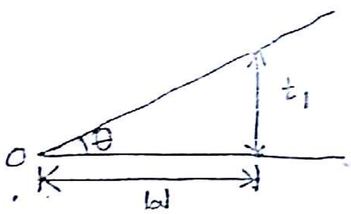
$$\delta = (2n-1)\pi \quad ; \quad n = 1, 2, 3 \cdots$$

$$\frac{2\pi}{\lambda}\left[2\mu t \cos(r+\theta)\right] - \pi = (2n-1)\pi$$

$$\Rightarrow \quad \boxed{2\mu t \cos(r+\theta) = n\lambda}$$

Expression for fringe width—

at the point O ($t=0$), the zero order dark fringe is obtained. Let when the thickness of film is t_1, we get first order dark fringe.



$$t = t_1 \quad , \quad n = 1$$

$$\Rightarrow \quad 2\mu t_1 \cos(r+\theta) = \lambda$$

$$\because \tan\theta = \frac{t_1}{\omega} \Rightarrow t_1 = \omega \tan\theta$$

$$2\mu \omega \cos(r+\theta) \tan\theta = \lambda$$

$$\boxed{\omega = \frac{\lambda}{2\mu \tan\theta \cos(r+\theta)}}$$

if $r = 0$ ~~and~~ and θ is very small, then

$\cos(r+\theta) \rightarrow \cos\theta = \cos 0^\circ = 1$

$\because$ θ is small, then $\tan\theta \rightarrow \theta$

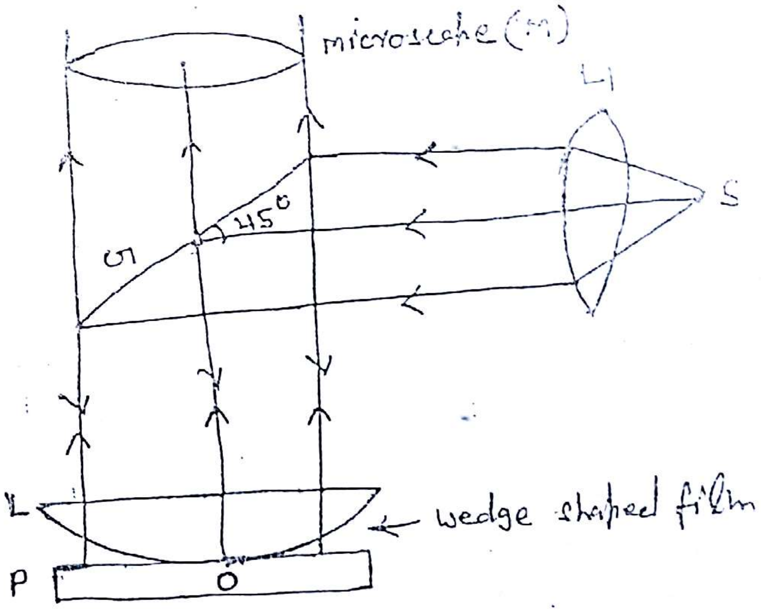
$$\Rightarrow \quad \boxed{\omega = \frac{\lambda}{2\mu\theta}}$$

Newton's Ring— The formation of Newton's rings is a special case of interference in an air film of variable thickness (wedge-shaped film). When a plane-convex lens of large focal length is placed on a plane glass plate, then a thin film of air is formed between the lower surface of the lens and the upper surface of the glass plate. When a monochromatic light is allowed to fall normally on the film, then circular fringes ~~are observed. In reflected light,~~ the centre of the circular fringes are dark followed by alternatively bright and dark circular rings and vice versa in transmitted light.

When white light is used in place of monochromatic light, a few coloured circular rings are seen around the point of contanct with violet colour and red colour outside in each ring.

Experimental arrangement—

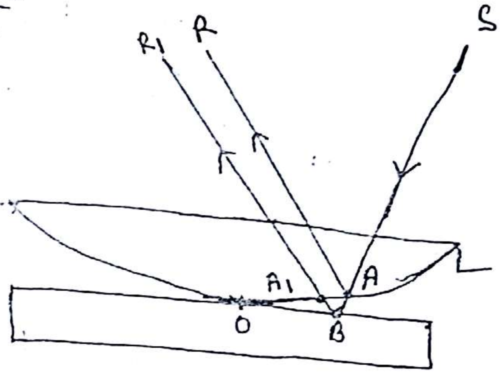
To observe the Newton's ring, the experimental arrangement is shown in the figure. S is a monochromatic source and is placed at the focus of a convex lens L_1. The horizontal parallel rays after the lens, fall on a glass plate G inclined at 45°. The rays are partly reflected from the inclined glass plate and fall normally on a plano-convex lens of large focal length L placed over a plane glass plate P.



The air film is formed in between the plano convex lens L and glass plate P arount the point of contact O. The interference takes place between the rays reflected from the upper and lower surfaces of the film and are viewed with a microscope (M) focussed on the air film.

The air film formed between the curved surface of the plano-convex lens and plane glass plate is of wedge shape.

- Formation of bright and dark rings— Light ray SA is incident on the upper surface of wedge shaped film. Some part of its amplitude is reflected and some part is refracted and this refracted part falls on the lower surface of film which is reflected again and we get the maximum and minimum intensity due to interference in between the waves AR and A_1R_1.



For max. intensity,

$$\cancel{2\mu t\cos(r+\theta) = (2n-1)\frac{\lambda}{2}; \; n = 1,2,3\ldots}$$

For min. intensity,

$$2\mu t\cos(r+\theta) = n\lambda \quad , \; n = 0,1,2,\ldots$$

For normal incidence, $r = 0$ and angle of wedge θ is very small.

so $\cos(r+\theta) \rightarrow \cos\theta \rightarrow 1$

$$\Rightarrow \; 2\mu t = (2n-1)\frac{\lambda}{2} \quad (\text{max. intensity}) \quad \text{—(1)}$$

$$\Rightarrow \; 2\mu t = n\lambda \quad (\text{min. intensity}) \quad \text{—(2)}$$

It is clear from equation (1) and (2) that for a bright or dark ring of any particular order, t should be constant. But here in the air film, the locus of points having the same thickness is a circle with its centre at the point of contact. Hence the fringes are circular rings having the common centre at the point of contact.

(i) - Diameters of bright and dark rings— Here L is a planoconvex lens placed on the glass plate P with O as the point of contact. Let R be the radius of curvature of curved surface of the lens and t is the thickness of the film at certain point Q.

In ΔCSQ,

$$(CQ)^2 = (CS)^2 + (SQ)^2$$

$$R^2 = (R-t)^2 + r_n^2$$

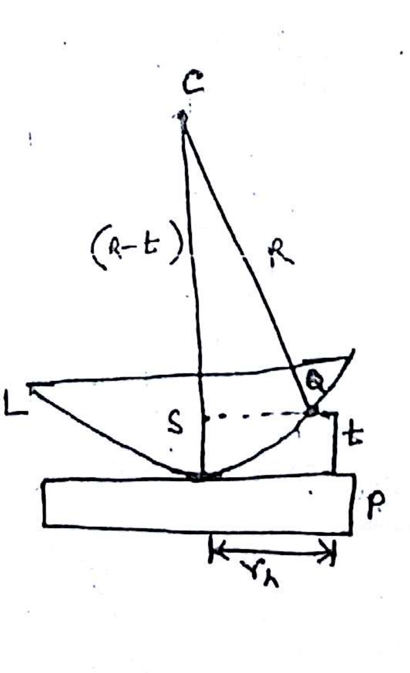
$$R^2 = R^2 + t^2 - 2Rt + r_n^2$$

$$2Rt = t^2 + r_n^2$$

As t is small, then t^2 is negligible.

$$2Rt = r_n^2$$

$$\boxed{t = \frac{r_n^2}{2R}}$$



For bright ring,

$$2\mu t = (2n-1)\frac{\lambda}{2} \Rightarrow \cancel{2}\mu \cdot \frac{r_n^2}{\cancel{2}R} = (2n-1)\frac{\lambda}{2}$$

$$\boxed{r_n^2 = (2n-1)\frac{\lambda R}{2\mu}}$$

$$\because D_n = 2r_n$$

$$\boxed{D_n^2 = 4r_n^2 = \frac{2(2n-1)\lambda R}{\cancel{2}\mu}} \qquad n = 1, 2, 3 \cdots$$

$$\Rightarrow D_n^2 \propto (2n-1)$$

$$\boxed{D_n \propto \sqrt{2n-1}}$$

Thus the diameters of the bright rings are proportional to the square roots of the odd natural numbers.

For dark ring -

$$\cancel{2}\mu \cdot \frac{r_n^2}{\cancel{2}R} = n\lambda \Rightarrow \boxed{r_n^2 = \frac{n\lambda R}{\mu}}$$

$$\because D_n = 2r_n \Rightarrow D_n^2 = 4r_n^2$$

$$\boxed{D_n^2 = \frac{4n\lambda R}{\mu}} \qquad n = 0, 1,$$

$$\Rightarrow D_n^2 \propto n$$

$$\boxed{D_n \propto \sqrt{n}}$$

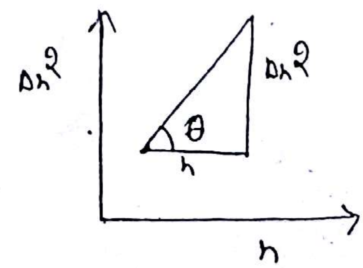
The diameters of successive dark rings are proportional to the square root of the natural numbers.

(iii) - Determination of wavelength of sodium light — In the experimental arrangement, S is a source of sodium light. The rings are viewed through travelling microscope M focussed on the air film. With the help of travelling microscope, the diameters of a number of dark rings are measured. If D_n is the diameter of n^{th} dark ring, then

$$D_n^2 = 4n\lambda R \quad (\text{for air } \mu = 1)$$

$$\frac{D_n^2}{n} = 4\lambda R \qquad \text{——— (3)}$$

If we plot a graph between n and D_n^2, we get a straight line. The slope of this straight line is $\frac{D_n^2}{n}$. Measuring this slope and R, wavelength of light, λ can be determined by equation (3).



If the contact of the lens and the plate at the centre is not perfect, the order n of the ring can not be known accurately due to uncertainty of the thickness of air film. In order to overcome the difficulty, the diameters of any two orders n and (n+p) of dark rings are measured and λ is calculated as follows —

For nth dark ring,

$$D_n^2 = 4n\lambda R \quad \text{--- (4)}$$

$$D_{n+p}^2 = 4(n+p)\lambda R \quad \text{--- (5)}$$

substracting,

$$D_{n+p}^2 - D_n^2 = 4p\lambda R$$

$$\boxed{\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}}$$

Here we calculate the diameters of dark rings in place of order (n). so that the above error eliminated, due to the imperfect contact at the centre.

(ii) - Determination of refractive index of liquid — The liquid whose refractive index (μ) is to be determined, is introduced between the glass plate and lens.

For nth dark ring,

$$D_n^2 = \frac{4n\lambda R}{\mu} \quad \text{--- (6)}$$

For (n+p)th dark ring,

$$D_{n+p}^2 = \frac{4(n+p)\lambda R}{\mu} \quad \text{--- (7)}$$

Substracting, $$(D_{n+p}^2 - D_n^2)_{liquid} = \frac{4p\lambda R}{\mu} \quad \text{--- (8)}$$

For air film,

$$(D_{n+p}^2 - D_n^2)_{air} = 4p\lambda R \quad \text{--- (9)}$$

Dividing, (9) / (8)

$$\boxed{\frac{(D_{n+p}^2 - D_n^2)_{air}}{(D_{n+p}^2 - D_n^2)_{liquid}} = \mu}$$

From this relation, we get the refractive index μ of a given liquid.

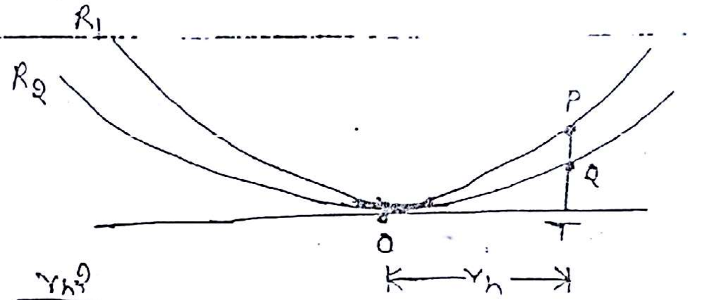
$\Rightarrow$ as $\mu > 1$, $(D_n)_{liquid} < (D_n)_{air}$

(v) Newton's rings formed by two curved surfaces—

A— When convex surface R_1 placed on the concave surface R_2—

If the two curved surfaces are placed in contact, instead of using a plane glass below the plane-convex lens, then Newton's rings are formed by interference between the light waves reflected from the upper and lower surfaces of the air film of varying thickness.

Let at the point P, the thickness of film is 't'.



$$PQ = t$$

$$\because PQ = PT - QT \quad \text{——— (10)}$$

$$\because PT = \frac{r_n^2}{2R_1}, \quad QT = \frac{r_n^2}{2R_2}$$

From (10),

$$t = \frac{r_n^2}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For bright ring—

$$2\mu t = (2n-1)\frac{\lambda}{2}$$

$$\cancel{2}\mu \cdot \frac{r_n^2}{\cancel{2}}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (2n-1)\frac{\lambda}{2}$$

$$\boxed{r_n^2 = \frac{(2n-1)\lambda}{2\mu\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}}$$

For dark ring—

$$2\mu t = n\lambda$$

$$\frac{\cancel{2}\mu \cdot r_n^2\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}{\cancel{2}} = n\lambda$$

$$\boxed{r_n^2 = \frac{n\lambda}{\mu\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}}$$

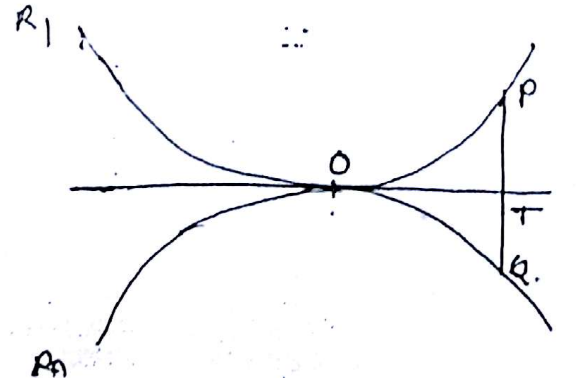
B— When one convex surface R_1 placed on the other convex surface R_2—

$$PQ = PT + QT$$

$$t = PT + QT \quad \text{——— (11)}$$

$$\because PT = \frac{r_n^2}{2R_1}$$

$$QT = \frac{r_n^2}{2R_2}$$



Putting in (11),

$$t = \frac{r_n^2}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \quad \text{—— (12)} \Rightarrow 2t = r_n^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

For bright ring ⇒

$$2\mu t = (2n-1)\frac{\lambda}{2} \qquad (\mu = 1 \text{ for air})$$

$$r_n^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{(2n-1)\frac{\lambda}{2}}{\mu}$$

$$\Rightarrow \boxed{r_n^2 = \frac{(2n-1)\frac{\lambda}{2}}{\mu\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}}$$

For dark ring —

$$r_n^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{n\lambda}{\mu}$$

$$\boxed{r_n^2 = \frac{n\lambda}{\mu\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}}$$

Diffraction

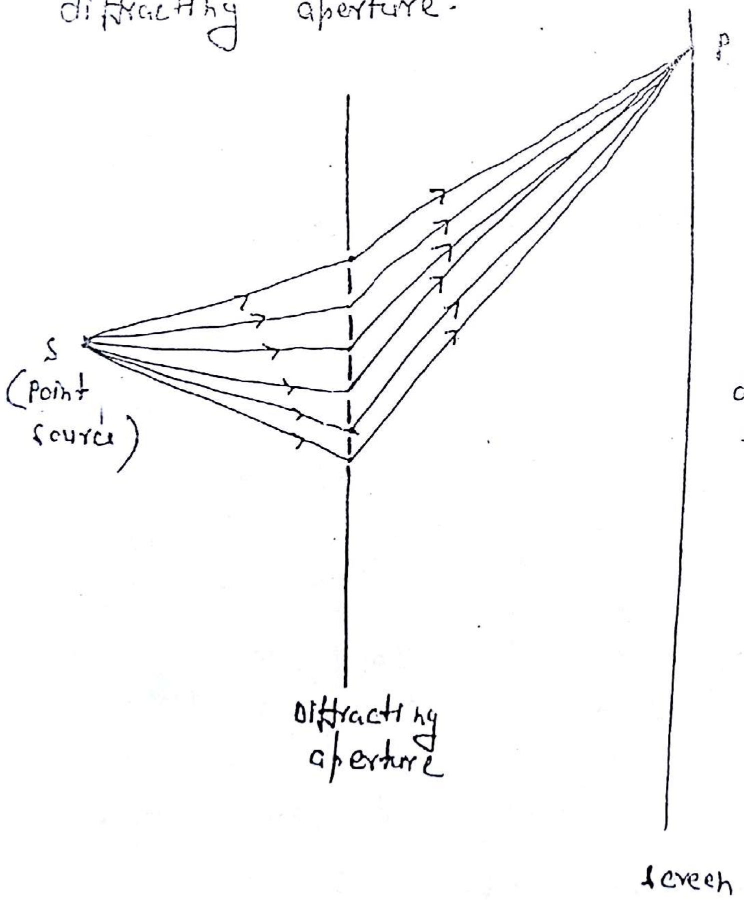
Diffraction - The departure of light path from true rectiline path or the bending of light round corners of an obstacle, is calle diffraction.

This is caused by the interference of innumerable secondry wavelets produced by the unobstructed portions of the same wave-front.

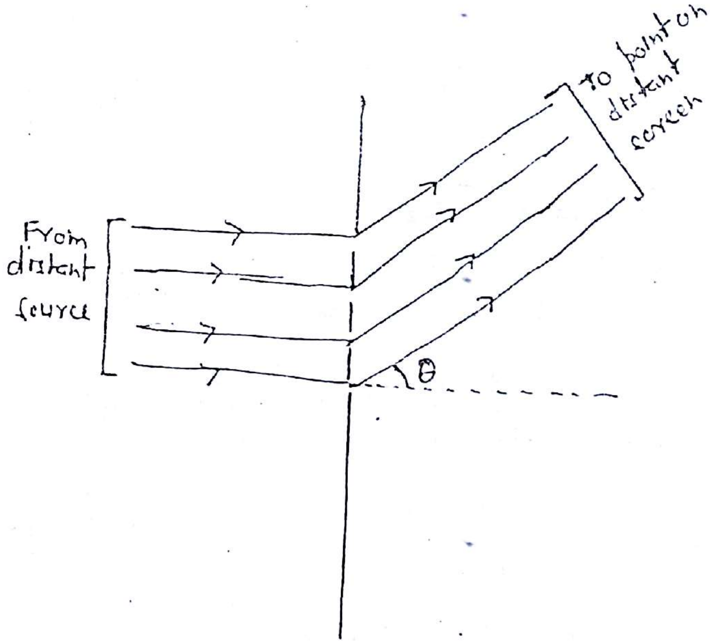
The diffraction phenomena are usually divided into two following classes.

1 - Fresnel diffraction
2 - Fraunhofer diffraction

In the Fresnel class of diffraction, the source of the light and the screen are generally at a finite distance from the diffracting aperture. In the Fraunhofer class of diffraction the source of light and screen are at infinite distance from the diffracting aperture.

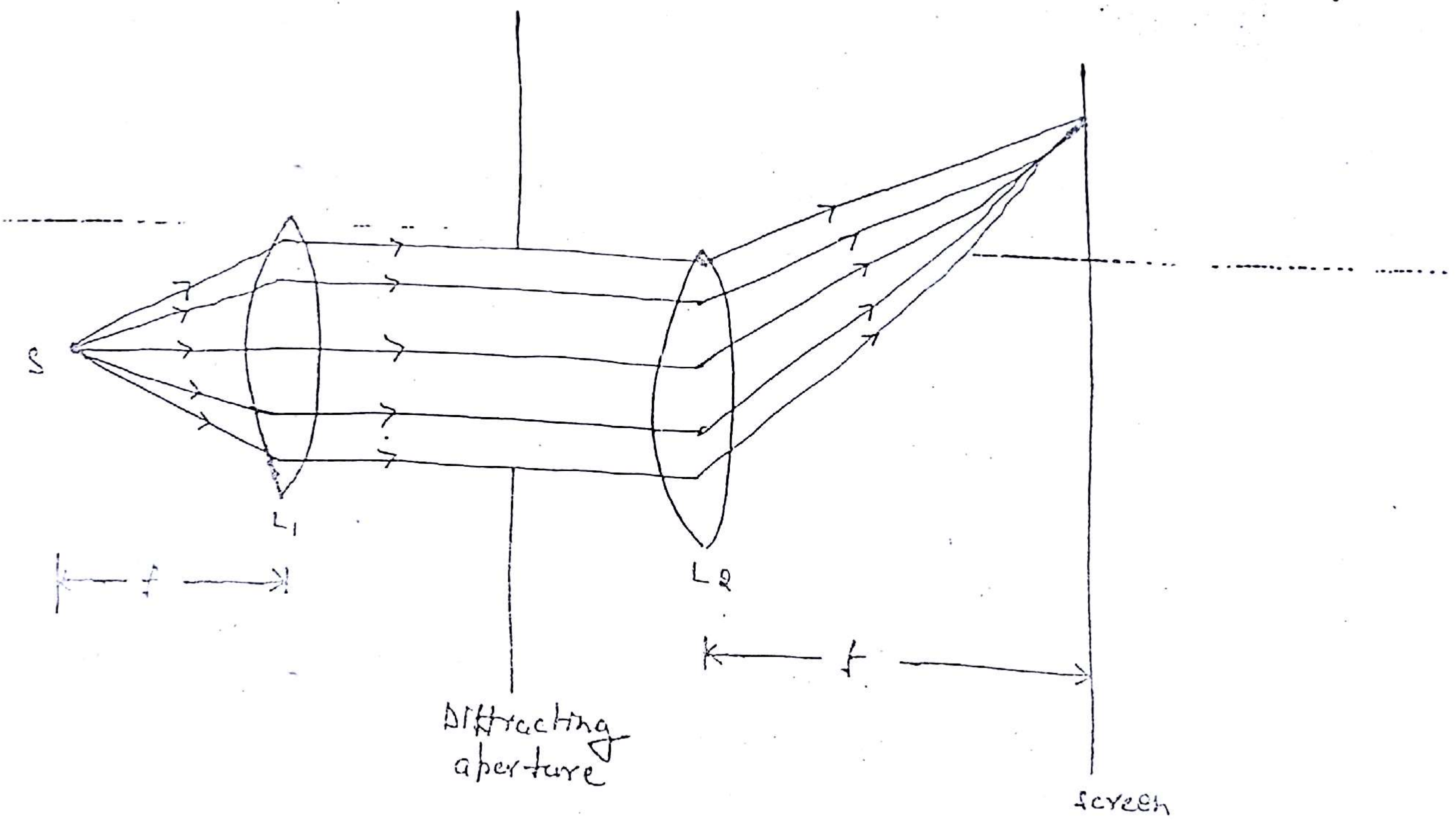


(Fresnel)



(Fraunhofer)

→ Fraunhofer diffraction can be easily achieved by placing the source on the focal plane of the convex lens and placing the screen on the focal plane of the another convex lens. This arrangement is as follows —



→ The differences between Fresnel diffraction and Fraunhofer diffraction are given below:

S.No.	Fresnel diffraction	Fraunhofer diffraction
1-	The source and the screen are at finite distance from the diffracting aperture.	The source and the screen are at infinite distance from the diffracting aperature.
2-	The wave fronts are divergent either spherical or cylindrical.	The wave fronts are plane which is realized by using convex lens.
3-	No mirrors or lenses are used for observation.	Diffracted light is collected by a lens as in a telescope.
4-	For obtaining Fresnel diffraction, zone plates are used.	For this, single, double slits or plane diffraction grating are used.
5-	The centre of diffraction pattern may be bright or dark depending upon the number of Fresnel zones.	The centre of diffraction pattern is always bright for all paths parallel to the axis of the lens.

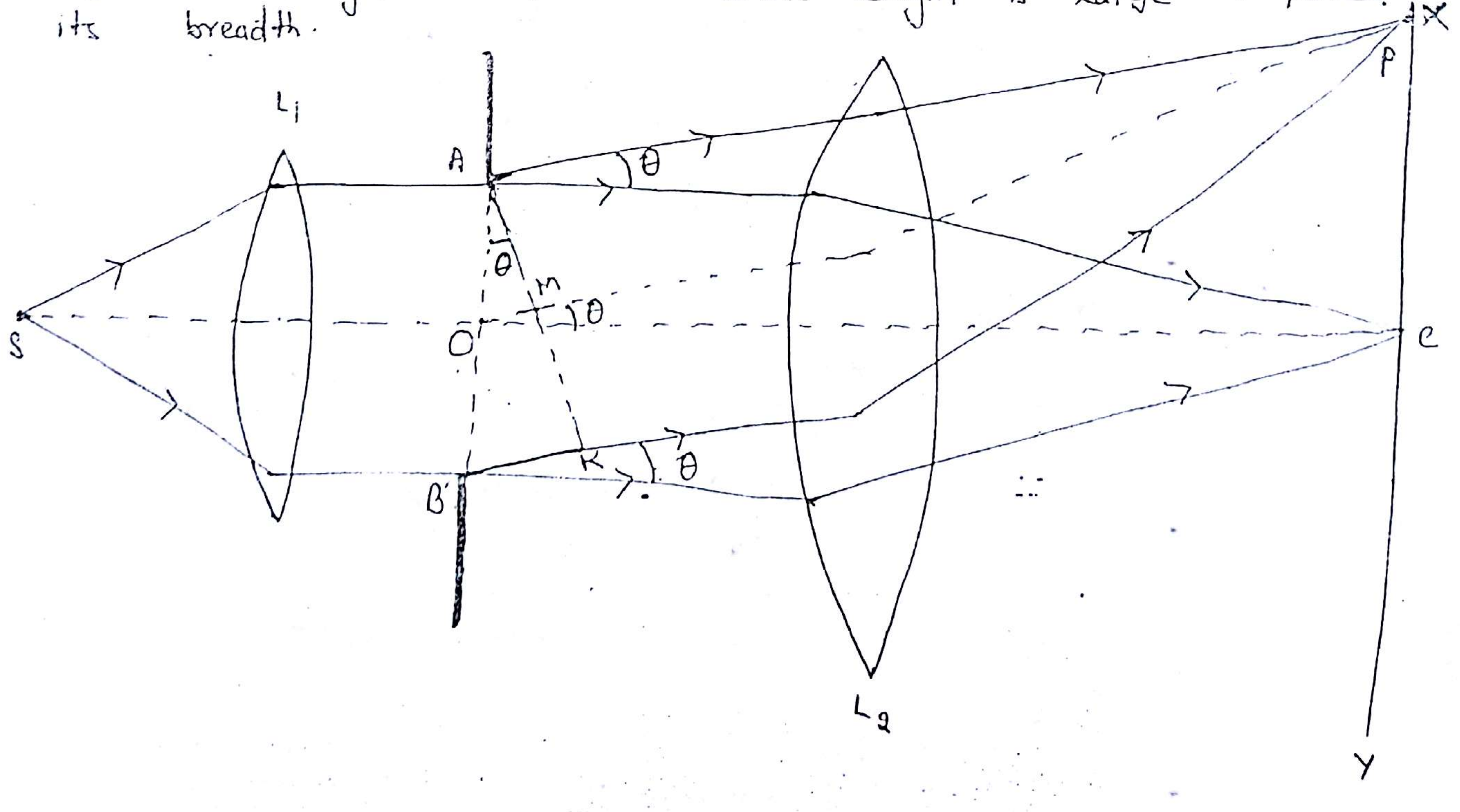
(2)

Fresnel diffraction is more general and it includes Fraunhofer diffraction as a special case. In this chapter we will study the simple diffraction phenomena i.e. F[illegible] diffraction.

→ Difference between interference and diffraction—

S.No.	Interference	Diffraction
1-	This phenomena is the result of interaction taking place between two separate wave fronts originating from two coherent sources.	1- This phenomena is the resu[lt] of interaction of light betw[een] the secondary wavelets originat[ing] from different points of the same wave front.
2-	The regions of minimum intensity are usually almost perfectly dark.	2- The regions of minimum intensity are not perfectly dark.
3-	Interference fringes are equally spaced	3- Diffraction fringes are ~~[illegible]~~ never equally spaced.
4-	All maxima are of same intensity	4- Maxima are of varying inten[sity]

Fraunhofer diffraction at a single slit— A slit is a rectangular aperture whose length is large compared to its breadth.



Let S is a slit source illuminated by monochromatic light of wavelength λ and L_1 is the collimating lens which is placed at a distance equal to the focal length of the lens from S. Thus the light emerged from lens L_1 as a parallel beam and falling on the slit AB of width 'e'. The diffracted light is focussed by another convex lens L_2 on the screen XY placed in the focal plane of lens L_2. The diffraction pattern consists of a bright band having alternate dark and bright bands of decreasing intensity on both the sides.

As there may be infinitely large number of point sources of secondary wavelets between A and B. Let us suppose that the aperture AB is divided into a large number of equal parts, each part being the source of secondary wavelets. The amplitude of vibration at P due to each part will be the same.

The resultant amplitude at P is given by,

$$R = \frac{A \sin\alpha}{\alpha} \quad \text{——— (1)}$$

where 2α = Phase difference between initial and final oscillations

Let us consider that the initial and final oscillations are producing due to the rays which are emerging from A and B. Hence

$$2\alpha = \frac{2\pi}{\lambda}(\text{Path difference})$$

$$= \frac{2\pi}{\lambda}(BK) = \frac{2\pi}{\lambda}(e\sin\theta)$$

$$\alpha = \frac{\pi}{\lambda}(e\sin\theta)$$

Intensity at P,

$$I \propto R^2$$

$$I \propto \frac{A^2 \sin^2\alpha}{\alpha^2}$$

$$I = \frac{I_0 \sin^2\alpha}{\alpha^2} \quad \text{——— (2)}$$

where $I_0 \propto A^2$

Central principal maxima — For the maximum intensity, the amplitude should be maximum.

$$R = \frac{A \sin\alpha}{\alpha} = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \cdots\right]$$

$$R = A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \cdots\right]$$

If $\alpha = 0$, then the maximum value of R is A. Then for $\alpha = 0$,

$$\frac{\pi}{\lambda}(e\sin\theta) = 0$$

$$\sin\theta = 0$$

$$\boxed{\theta = 0}$$

Hence the point, at which parallel rays meets, is the central principal maxima.

Secondry minima — For minima, intensity should be zero. From eq. (2), I is zero, when $\sin\alpha = 0$

$$\alpha = 0, \pm\pi, \pm 2\pi, \ldots$$

$\alpha \neq 0$, because this is the condition of maximum intensity. Hence

$$\alpha = \pm\pi, \pm 2\pi, \ldots$$

$$\alpha = \pm n\pi$$

where $n = 1, 2, 3, \ldots$

$$\frac{\cancel{\pi}}{\lambda}(e\sin\theta) = \pm n\cancel{\pi}$$

$$\boxed{e\sin\theta = \pm n\lambda}$$

3- **Secondry Maxima** — The condition of secondry maxima is obtained by diff. eq. (2) w.r.to α and equal to zero

$$\Rightarrow \frac{dI}{d\alpha} = 0$$

$$\frac{d}{d\alpha}\left(\frac{I_0 \sin^2\alpha}{\alpha^2}\right) = 0$$

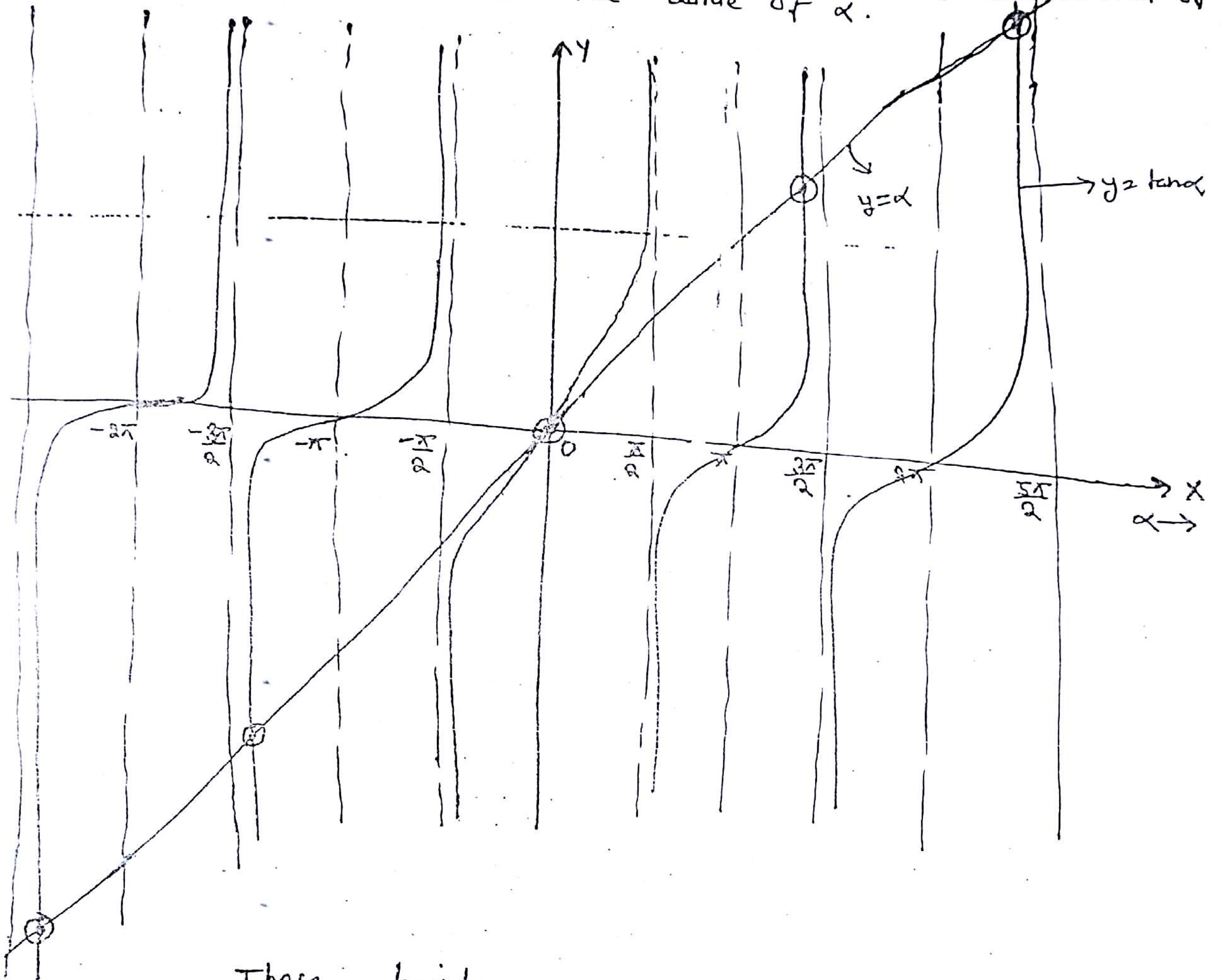
$$I_0 \frac{d}{d\alpha}\left(\frac{\sin\alpha}{\alpha}\right)^2 = 0$$

$$2 I_0 \frac{\sin\alpha}{\alpha} \cdot \frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$$

$$\alpha\cos\alpha - \sin\alpha = 0$$

$\boxed{\alpha = \ldots}$

The eq. (3) can be solved by graphreally by ploting the curves $y = \alpha$ and $y = \tan\alpha$. The points of intersection of these two curves gives the value of α.



These points corresphond to the value of

$$\alpha = 0, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$$

At $\alpha = 0$ (central principal maxima)

Hence $\alpha \neq 0$

$$\Rightarrow \alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots = \pm(2n+1)\frac{\pi}{2}$$

Intensity at first secondry maxima, (let $I_0 = 1$)

$$I_1 = \left(\frac{\sin\frac{3\pi}{2}}{\left(\pm\frac{3\pi}{2}\right)}\right)^2 = \frac{4}{9\pi^2}$$

Intensity at second secondry maxima,

$$I_2 = \left(\frac{\sin\frac{5\pi}{2}}{\pm\frac{5\pi}{2}}\right)^2 = \frac{4}{25\pi^2}$$

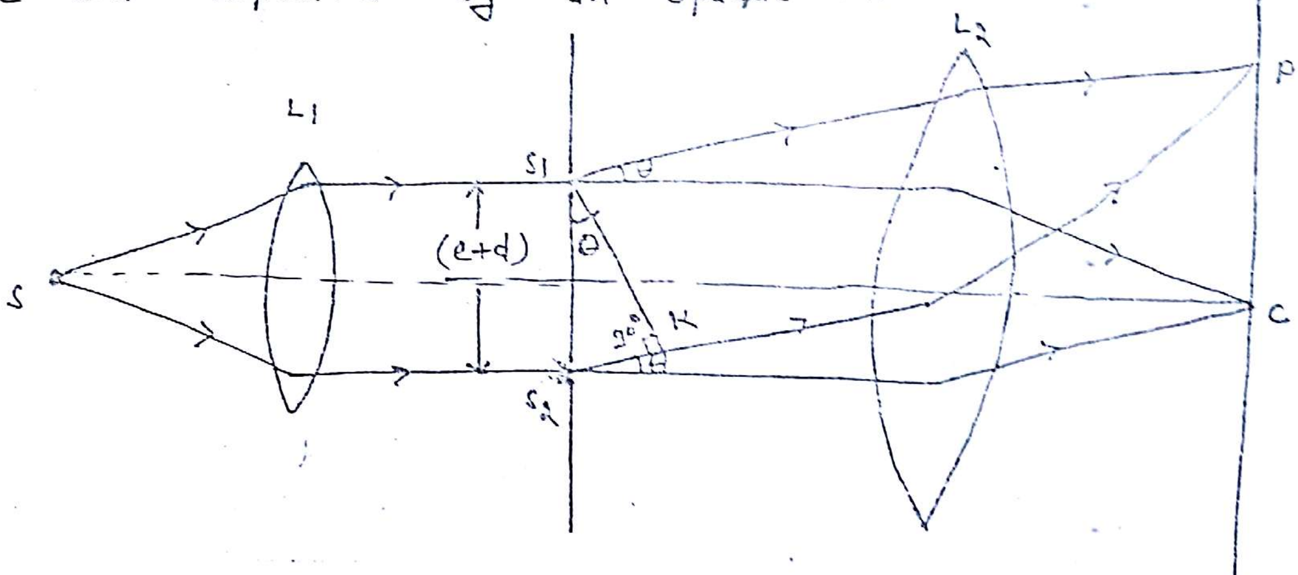
Intensity at third secondary maxima, $I_3 = \dfrac{\sin^2\left(\pm\frac{7\pi}{2}\right)}{\left(\pm\frac{7\pi}{2}\right)^2}$

$$= \frac{4}{49\pi^2}$$

The ratio of these intensities are,

$$I_0 : I_1 : I_2 : I_3 : \ldots = 1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2}$$

Diffraction at two slit — Let a plane wavefront be incident normally on two slits AB and CD of equal width e and separated by an opaque distance d.



The diffracted light is focussed on the screen XY. Suppose each slit diffracts the beam in a direction making an angle θ with the direction of incident beam. Due to a single slit, the resultant amplitude is $\frac{A\sin\alpha}{\alpha}$. Now consider that the two slits equivalent to two coherent sources placed at the middle points S_1 and S_2 of the slits and each sending of amplitude $\frac{A\sin\alpha}{\alpha}$ in a direction θ.

The resultant amplitude at point P will be the result of interference between two waves of same amplitude $\frac{A\sin\alpha}{\alpha}$ and a phase difference δ.

$\therefore$ Path difference $(S_2K) = (e+d)\sin\theta$

Phase difference $(\delta) = \frac{2\pi}{\lambda} \times (S_2K)$

$$\delta = \frac{2\pi}{\lambda}(e+d)\sin\theta$$

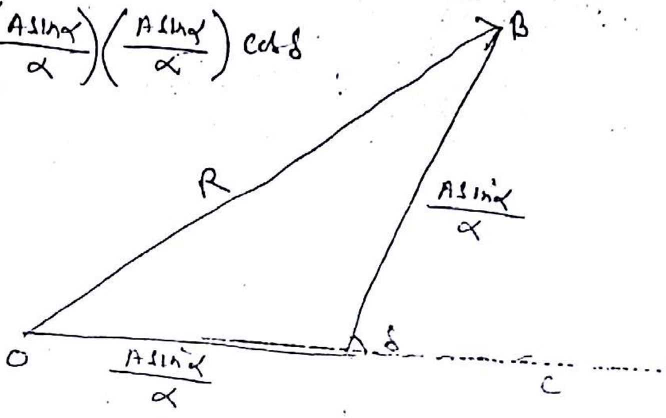
The resultant amplitude (R) at point P,

$$(OB)^2 = (OA)^2 + (OB)^2 + 2(OA)(OB)\cos\delta$$

$$R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2 + \left(\frac{A\sin\alpha}{\alpha}\right)^2 + 2\left(\frac{A\sin\alpha}{\alpha}\right)\left(\frac{A\sin\alpha}{\alpha}\right)\cos\delta$$

$$= \frac{2A^2\sin^2\alpha}{\alpha^2}\left(1+\cos\delta\right)$$

$$= \frac{2A^2\sin^2\alpha}{\alpha^2} \cdot 2\cos^2\left(\frac{\delta}{2}\right)$$



$$R^2 = \frac{4A^2\sin^2\alpha \cdot \cos^2\beta}{\alpha^2}$$

where $\beta = \frac{\delta}{2} = \frac{\pi}{\lambda}(e+d)\sin\theta$

$$\Rightarrow \quad I \propto R^2$$

$$I \propto \frac{4A^2\sin^2\alpha}{\alpha^2} \cdot \cos^2\beta$$

Let $R_0^2 \propto A^2$

$$I = 4R_0^2\frac{\sin^2\alpha}{\alpha^2} \cdot \cos^2\beta \quad \text{——— (2)}$$

In equation (2), the term $\frac{R_0^2\sin^2\alpha}{\alpha^2}$ gives diffraction pattern due to each slit and $\cos^2\beta$ gives interference pattern due to diffracted light waves from the two slits.

Due to single slit diffraction, at $\alpha = 0$ or $\theta = 0$, central principal maxima will be obtained.

For secondary minima,

$$\alpha = \pm n\pi$$

$$\Rightarrow \boxed{e\sin\theta = \pm n\lambda} \qquad \text{where } n = 1, 2, 3 \ldots$$

For secondary maxima,

$$\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$$

$$\alpha = \pm(2n+1)\frac{\pi}{2}$$

$$\Rightarrow \boxed{e\sin\theta = \pm(2n+1)\frac{\lambda}{2}} \qquad \text{where } n = 1, 2, 3 \ldots$$

2 - The term $\cos^2\beta$ gives the interference and in the intensity pattern gives a set of equidistant dark and bright fringes.

For bright fringes,

$$\cos^2\beta = 1$$

$$\cos\beta = \pm 1$$

$$\beta = 0, \pm\pi, \pm 2\pi, \ldots$$

$$\beta = \pm m\pi$$

where $m = 0, 1, 2 \ldots$

$$\frac{\cancel{\pi}}{\lambda}(e+d)\sin\theta = \pm m\cancel{\pi}$$

$$\boxed{(e+d)\sin\theta = \pm m\lambda}$$

For dark fringes,

$$\cos^2\beta = 0$$

$$\cos\beta = 0$$

$$\beta = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$$

$$\beta = \pm(2m+1)\frac{\pi}{2}$$

where $m = 0, 1, 2 \ldots$

$$\frac{\cancel{\pi}}{\lambda}(e+d)\sin\theta = \pm(2m+1)\frac{\cancel{\pi}}{2}$$

$$\boxed{(e+d)\sin\theta = \pm(2m+1)\frac{\lambda}{2}}$$

Missing order in double slit diffraction —

For bright fringes in interference,

$$(e+d)\sin\theta = \pm m\lambda \quad \text{—— (1)}$$

For secondary minima, in diffraction,

$$e\sin\theta = \pm n\lambda \quad \text{—— (2)}$$

eqn. (1) / eq. (2)

$$\frac{e+d}{e} = \frac{m}{n} \quad \text{—— (3)}$$

The nature of the diffraction pattern due to the double slits depends upon the relative values of e and d. If, however, e is kept constant and d is varied, then certain orders of interference maxima will be missing.

1 - If $e = d$, from eq. (1)

$m = 2n$

$\therefore n = 1, 2, 3, \cdots$

$\Rightarrow m = 2, 4, 6, \cdots$

Hence 2nd, 4th, 6th, ... order interference maxima will be missing.

2 - if $e = \frac{d}{2}$, from eqn (1),

$m = 3n$

$\therefore n = 1, 2, 3 \cdots$

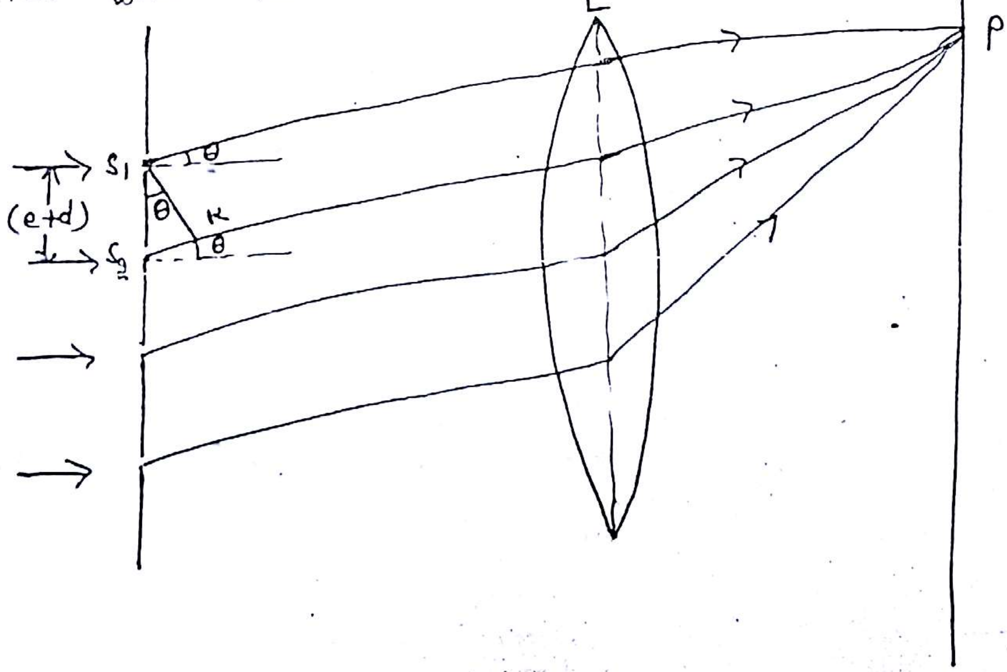
$\Rightarrow m = 3, 6, 9, \cdots$

Thus 3rd, 6th and 9th .... order interference maxima will be missing.

Diffraction at N slit or transmission grating—

A diffraction grating consists of a large number of parallel slits of equal width and separated from one another by equal opaque spaces.

It may be constructed by ruling a large number of parallel and equidistant lines on a plane glass plate with the help of a diamond point. The replicas of the original grating are prepared by pouring a thin layer of collidion solution over it and then allowed to hardened. This layer is then removed from the original grating and fixed between two glass plates which serves as a plane transmission grating.



Let a plane wavefront of monochromatic light be incident normally on N- parallel slits of the grating. Each point in the slits then sends out secondary wavelets in all directions.

The resultant amplitude due to a single slit,

$$R = \frac{A \sin\alpha}{\alpha}$$

Path diff. $= SK = (e+d) \sin\theta$

Phase difference $(\delta) = \frac{2\pi}{\lambda}(e+d) \sin\theta$.

Where $(e+d)$ is called grating element.

When the n waves of 'a' amplitude focused on a point then the amplitude of resultant oscillation is

$$R = \frac{a \sin\frac{n\delta}{2}}{\sin\frac{\delta}{2}}$$

where $a = \frac{A \sin\alpha}{\alpha}$

if, $n = N$; $\delta = \frac{2\pi}{\lambda}(e+d)\sin\theta$

$\frac{\delta}{2} = \frac{\pi}{\lambda}(e+d)\sin\theta$

$$\Rightarrow \boxed{R = \frac{A \sin\alpha}{\alpha} \cdot \frac{\sin N\beta}{\sin\beta}}$$

$\frac{\delta}{2} = \beta$

Resultant intensity at P,

$$I = R^2 = \frac{A^2 \sin^2\alpha}{\alpha^2} \cdot \frac{\sin^2 N\beta}{\sin^2\beta} \quad \text{---} (1)$$

The first term $\frac{A^2\sin^2\alpha}{\alpha^2}$ represents the diffraction pattern due to single slit. Second term $\frac{\sin^2 N\beta}{\sin^2\beta}$ represents the interference pattern due to N slits.

1- Central principal maxima—

For maximum intensity,

$$\sin\beta = 0$$

$$\beta = 0, \pm\pi, \pm 2\pi, \ldots$$

$$\beta = \pm n\pi \quad \text{where } n = 0, 1, 2 \ldots$$

$$\therefore \frac{\cancel{\pi}}{\lambda}(e+d)\sin\theta = \pm n\cancel{\pi}$$

$$\boxed{(e+d)\sin\theta = \pm n\lambda}$$

For $\beta = \pm n\pi$, we will get central principal maxim

$$\Rightarrow \frac{\sin N\beta}{\sin\beta} = \frac{\sin N(\pm n\pi)}{\sin(\pm n\pi)} = \frac{0}{0} = \text{indeterminate term}$$

1. Then,

$$\lim_{\beta \to \pm n\pi} \frac{\sin N\beta}{\sin\beta} = \lim_{\beta \to \pm n\pi} \frac{N\cos N\beta}{\cos\beta} = \frac{N\cos(\pm n\pi)}{\cos(\pm n\pi)} = \pm N$$

From eqn (1),

$$I = \frac{N^2 A^2 \sin^2\alpha}{\alpha^2}$$

This is the intensity at central principal

2 -

Secondry minima —

For zero intensity,

$$\sin N\beta = 0$$

$$N\beta = \pm m\pi$$

$$N \cdot \frac{\pi}{\lambda}(e+d)\sin\theta = \pm m\pi$$

$$\boxed{(e+d)\sin\theta = \pm \frac{m\lambda}{N}}$$

Where m can take all integral values except 0, N, 2N - - - nN because for these points, we will get the central principal maxima.

3 - Secondry maxima —

$$\frac{dI}{d\beta} = 0$$

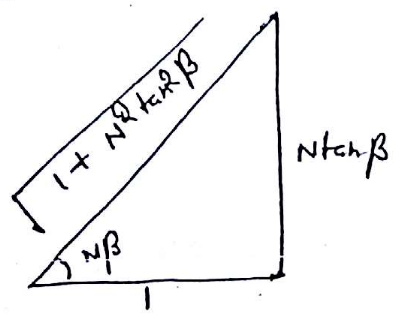
$$\frac{d}{d\beta}\left[\frac{A^2\sin^2\alpha}{\alpha^2}\left(\frac{\sin N\beta}{\sin\beta}\right)^2\right] = 0$$

$$= \frac{A^2\sin^2\alpha}{\alpha^2} \cdot 2\frac{\sin N\beta}{\sin\beta} \cdot \frac{\sin\beta\, N\cos N\beta - \cos\beta \cdot \sin N\beta}{\sin^2\beta} = 0$$

$$\sin\beta\, N\cos N\beta - \cos\beta \sin N\beta = 0$$

$$\sin\beta\, N\cos N\beta = \cos\beta \sin N\beta$$

$$\boxed{\tan N\beta = N\tan\beta}$$



$$\Rightarrow \quad \sin N\beta = \frac{N\tan\beta}{\sqrt{1+N^2\tan^2\beta}}$$

$$\frac{\sin N\beta}{\sin\beta} = \frac{1}{\sin\beta} \cdot \frac{\frac{N\sin\beta}{\cos\beta}}{\sqrt{1+\frac{N^2\sin^2\beta}{\cos^2\beta}}}$$

$$\frac{\sin N\beta}{\sin\beta} = \frac{N}{\sqrt{\cos^2\beta + N^2\sin^2\beta}} = \frac{N}{\sqrt{1-\sin^2\beta + N^2\sin^2\beta}}$$

$$= \frac{N}{\sqrt{1+(N^2-1)\sin^2\beta}}$$

From eqn (1),

$$\boxed{I = \frac{A^2\sin^2\alpha}{\alpha^2} \cdot \frac{N^2}{1+(N^2-1)\sin^2\beta}}$$

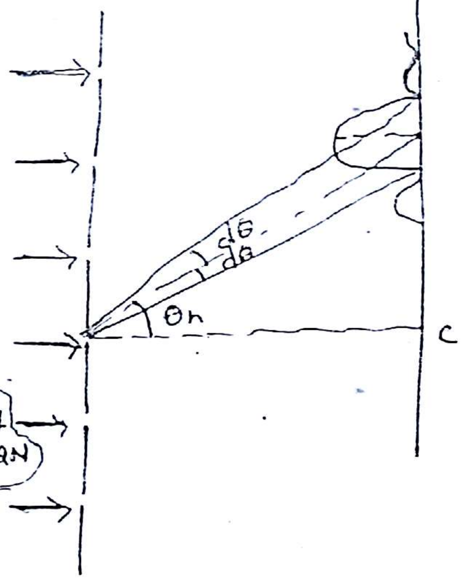
⇒ Angular width of central principal maxima —

The nth order principal maxima is obtained in the direction θ_n, given by

$(e+d)\sin\theta_n = n\lambda$ ——— (1)

For secondary minimais

$(e+d)\sin\theta_n = \frac{m\lambda}{N}$ where m = Integer; except 0, N, 2N



Let the first minima adjacent to the nth (principal) maxima be obtained in the direction $(\theta_n \pm d\theta)$ where $d\theta$ is called the angular half width of the nth order principal maxima

$$\Rightarrow \quad m = nN \pm 1$$

Now, $(e+d)\sin(\theta_n \pm d\theta) = (nN \pm 1)\frac{\lambda}{N}$ ——— (2)

eq. (2) / eq. (1)

$$\frac{\sin(\theta_n \pm d\theta)}{\sin\theta_n} = (nN\pm1)\frac{1}{nN}$$

$$\frac{\sin\theta_n\cos d\theta \pm \cos\theta_n\sin d\theta}{\sin\theta_n} = (nN\pm1)\frac{1}{nN}$$

$$\cos d\theta \pm \sin d\theta\cot\theta_n = 1 \pm \frac{1}{nN}$$

if $d\theta$ is small, then

$\cos d\theta = 1$, $\sin d\theta = d\theta$

$\Rightarrow \cancel{1} \pm d\theta\cot\theta_n = \cancel{1} \pm \frac{1}{nN}$

$$\boxed{d\theta = \frac{1}{nN\cot\theta_n}}$$

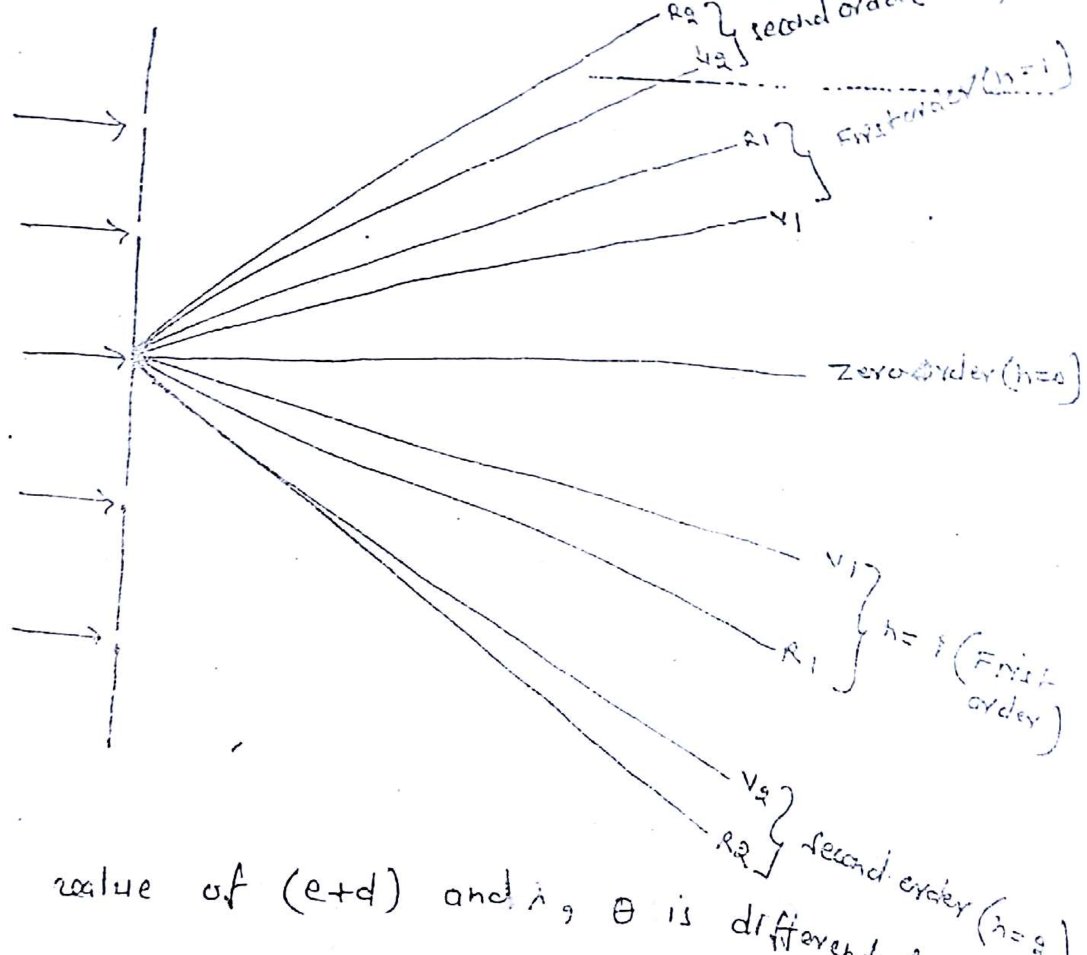
Formation of spectra by grating—

The principal maxima are given by

$$(e+d)\sin\theta = \pm n\lambda \quad \text{--- (1)}$$

The following conclusion drawn from the above equation,

1- For fixed values of $(e+d)$ and n, θ is different for different wavelength. As $\lambda_r > \lambda_v$, then $\theta_r > \theta_v$.



- For fixed value of $(e+d)$ and λ, θ is different for different order n.
- For $n=0$, $\theta=0$, for all values of λ. Hence, the zeroth order principal maxima is always white.
- Most of the intensity of light is concentrated in zeroth order principal maxima and decreases with increasing order.

Determination of wavelength of light—

For n^{th} order principal maxima,

$$(e+d)\sin\theta = \pm n\lambda$$

$$\boxed{\lambda = \frac{(e+d)\sin\theta}{\pm n}}$$

Thus, the determination of λ, involves the measurement of $(e+d)$, the grating element, θ the angle of diffraction and n, the order of spectrum.