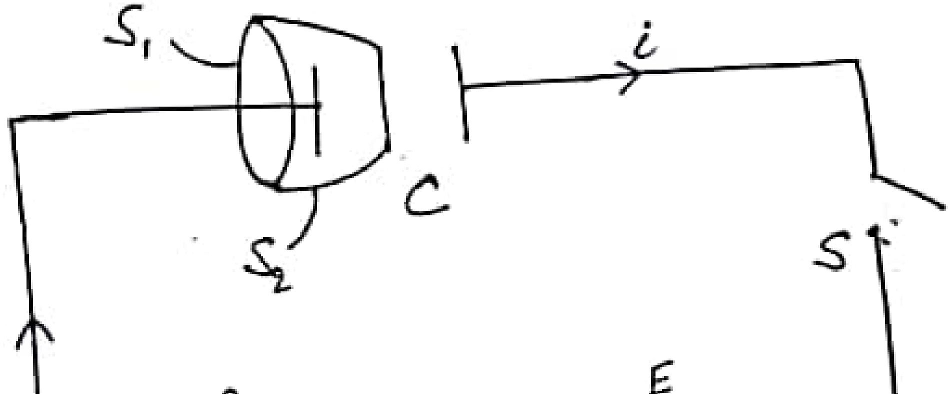
Displacement current→
The concept of displacement current was first introduced by Maxwell purely on theoretical ground. Maxwell postulated that it is not only the current in a conductor that produces a magnetic field but a changing electric field in a vaccum or a dielectric also produces a magnetic field. It means that changing electric field is equivalent to a current which flows as long as the electric field is changing. This equivalent current produces the same magnetic effect as an ordinary current in a conductor. This equivalent current is known as displacement current.

Modified Ampere's law→



To establish the concept of displacement current let us consider the process of charging of parallel plate capacitor through a series circuit containing cell E, resistor R, a capacitor C and switch S as shown in fig. When the switch S is pressed circuit is closed and a current flows in the circuit as a result of which charge starts accumulating gradually on the plates of the capacitor. The

Current in the circuit increases as the charge grows on the plates. When the capacitor is fully charged to the e.m.f of the cell, the current stops. There is no actual flow of charge between the plates during charging. If we place a compass needle in the space between the plates, the needle deflects. This indicates that there is a magnetic field b/w the capacitor plates even though there is no motion of charge. This indicates that there must be some other source of magnetic field in the gap. The other source is nothing but the charging electric field b/w the plates.

Let us consider a plane surface S_1 and a hemispherical surface S_2 around a capacitor plate as shown in fig. Let both the surfaces are bounded by the same closed path l. Now apply Ampere's circuital law to both surfaces. If Ampere's law is applied to surface S_1, then

$$\oint_{S_1} \vec{B}\cdot\vec{dl} = \mu_0 I \quad \text{——— (1)}$$

Because during the process of charging, current i has been flowing through plane surface S_1. But if it is applied to the hemispherical surface S_2, then

$$\oint_{S_2} \vec{B}\cdot\vec{dl} = 0 \quad \text{——— (2)}$$

Eq^n (1) and (2) contradict each other and thus both cannot be correct. Maxwell remove this discrepancy by adding one more term on the RHS of eq^n (1). This new term is based on ideology that a charging electric field is a source of magnetic field in the gap b/w the capacitor plates and is equivalent to displacement current that is denoted by i_d. The condition of continuity of the current requires the displacement current is to be equal to $\epsilon_0 \frac{d\phi_E}{dt}$, where

ϕ_E = flux of the electric field through an area bounded by the closed curve

ϵ_0 = permittivity of the free space

$$i_d = \epsilon_0 \frac{d\phi_E}{dt} \quad \text{——} (3)$$

We now show that the displacement current i_d in the gap b/w the plates is equal to the conduction current i flowing in the connecting wire. Suppose q be the charge collected on the capacitor plates at any instant t, then the electric field developed b/w the plates will be→

$$E = \frac{q}{\epsilon_0 A} \quad \text{——} (4)$$

Here A is the surface area of each plate.

Differentiating eq^n (4), w.r.t 't' we get→

$$\frac{dE}{dt} = \frac{1}{\epsilon_0 A}\frac{dq}{dt} = \frac{1}{\epsilon_0 A} i$$

$$i = \epsilon_0 A \frac{dE}{dt} \quad \text{——} (5)$$

By def. the displacement current i_d is→

$$i_d = \epsilon_0 \frac{d\phi_E}{dt} \quad \text{——} (6)$$

But we know that→ $\phi_E = EA$

$$i_d = \epsilon_0 A \frac{dE}{dt} \quad \text{——} (7)$$

From eq^n (5) and (7) it is clear that

$$i_d = i$$

Eq^n (7) may also be written as

$$i_d = A\frac{dD}{dt} \quad (\because D = \epsilon_0 E) \quad \text{——} (8)$$

In terms of displacement current density

$$\frac{i_d}{A} = \frac{dD}{dt}$$

$$J_d = \frac{dD}{dt} \quad \text{——} (9)$$

The total current is the sum of conductor current and displacement current. The conduction current flow through the resistance part of the circuit and displacement current through pure capacitor part. Thus the total current

$$I = i_R + i_d$$

$$\frac{I}{A} = \frac{i_R}{A} + \frac{i_d}{A}$$

$$J = J_R + J_d$$

where J_R and ~~d~~ J_d are conduction and displacement current.

Characteristics of displacement current →

1 Displacement current is a current in the sense that it produces a magnetic field. It has a finite value even in perfect vaccum where there is no charge at all.

2 Displacement current serves the purpose to make the total current across the discontinuity in a conduction current.

3 Displacement current in good conductors is almost nil as compared to conduction current at frequencies less than optical freq. ($\simeq 10^{15}$ Hz)

Eqn of Continuity → Eqn of continuity is based upon the law of conservation of charge. Acc. to which charge can be neither created nor destroyed.

Let us consider any region of volume V, bounded by closed surface S. The current flow through the closed surface →

$$-\frac{dq}{dt} = i = \oint_S \vec{J}\cdot\vec{ds} \text{ ——— (1)}$$

From the conservation principle, this current flow through a region, must be balanced by the rate of decrease of charge flowing through that region, that is →

$$i = -\frac{dq}{dt} = \oint_S \vec{J}.\vec{ds}$$

But $q = \int \rho dv$ where ρ = Vol. charge distribution

$$\oint_S \vec{J}.\vec{ds} + \int_V \frac{\partial \rho}{\partial t} dv = 0 \text{ ——— (2)}$$

But from gauss divergence theorem →

$$\oint_S \vec{J}.\vec{ds} = \int_V \operatorname{div} J \, dv = \int_V \vec{\nabla}.\vec{J} \, dv$$

Thus

$$\int_V \left(\left(\vec{\nabla}.\vec{J} + \frac{\partial \rho}{\partial t}\right)\right) dv = 0 \quad \text{or} \quad \vec{\nabla}.\vec{J} + \frac{\partial \rho}{\partial t} = 0 \text{ ——— (3)}$$

This is the eqn of continuity, expressing the fact that the electric charges can neither be created nor destroyed

For static fields, $\frac{\partial \rho}{\partial t} = 0$ ——— (4)

$$\vec{\nabla}.\vec{J} = 0$$

Q1 A parallel plate capacitor is being charged. Show that the displacement current across an area in the region between the plates and parallel to it is equal to the conduction current in the connecting wires.

Sol^n → The electric field across an area A in the region between the plates is given by

$$E = \frac{Q}{\epsilon_0 A}$$

where Q is the charge accumulated at the positive plate. The flux of the field through an area A is

$$\phi_E = E \cdot A = \frac{Q}{\epsilon_0 A} \cdot A = \frac{Q}{\epsilon_0}$$

The displacement current is given by

$$i_d = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 \frac{d}{dt}\left(\frac{Q}{\epsilon_0}\right) = \frac{dQ}{dt}$$

But $\frac{dQ}{dt}$ is the rate at which charge is transported to the positive plate through the connecting wires, that is, it is the conduction current.

Therefore, $i_d = i$

Q2 In a material for which $\sigma = 5\,S/m$ and $\epsilon_r = 1$, the electric field intensity is $E = 250 \sin(10^{10} t)$ V/m. Find conduction and displacement current densities and the frequency at which both have equal amplitude.

Sol^n → The conduction current density $J = \sigma E$

Here $\sigma = 5\,S/m$ and $E = 250 \sin(10^{10} t)$

$$J_c = 5 \times 250 \sin(10^{10} t) = 1250 \sin(10^{10} t)\ A/m^2$$

The displacement current density, $J_d = \epsilon \frac{\partial E}{\partial t}$

$$J_d = \epsilon \frac{\partial}{\partial t}\left[250\sin(10^{10}t)\right]$$

$$= \epsilon_0 \epsilon_r 250 \times 10^{10} \cos(10^{10}t)$$

$$J_d = 8.85 \times 10^{-12} \times 1 \times 250 \times 10^{10} \cos(10^{10}t)$$

$$= 22.125 \cos(10^{10}t) \; A/m$$

For harmonically time varying field, $J_d = j\omega\epsilon E$

If 'f' is the frequency at which $|\vec{J_d}| = |\vec{J_c}|$.

$$\sigma E = \omega \epsilon E$$

$$\omega = \frac{\sigma}{\epsilon} = \frac{\sigma}{\epsilon_0 \epsilon_r} = 2\pi f$$

$$f = \frac{\sigma}{2\pi\epsilon} = \frac{5}{2 \times 3.14 \times 8.85 \times 10^{-12}}$$

$$= 8.996 \times 10^{10} \; Hz$$

Q1 Show that eqⁿ of continuity $\text{div}\,\vec{J} + \frac{\partial \rho}{\partial t} = 0$ is contained in Maxwell's eqⁿ.

or

Starting from Maxwell's eqⁿˢ, establish the eqⁿ of continuity.

Solⁿ → Acc. to Maxwell's fourth eqⁿ

$$\text{curl}\,H = J + \frac{\partial D}{\partial t}$$

Taking divergence of both sides, we get

$$\text{div}(\text{curl}\,H) = \text{div}\left(J + \frac{\partial D}{\partial t}\right)$$

But $\text{div}\,\text{curl}\,H = 0$

$$\text{div}\left(J + \frac{\partial D}{\partial t}\right) = 0$$

$$\text{div}\,J + \text{div}\left(\frac{\partial D}{\partial t}\right) = 0$$

$$\text{div}\,J + \frac{\partial}{\partial t}(\text{div}\,D) = 0$$

From Maxwell's first eqⁿ, $\text{div}\,D = \rho$ where ρ is the charge density

$$\therefore \text{div}\,J + \frac{\partial \rho}{\partial t} = 0$$

Maxwell eqn in differential form!

Maxwell first eqn (Gauss law for electrostatics) → Acc. to which divergence of electric displacement vector is equal to the vol. charge density ρ.
In differential form it can be expressed as

$$\operatorname{div} \vec{D} = \rho \text{ ——— } (1)$$

$\vec{D}$ = Displacement vector
ρ = Vol. charge density.

Proof → When a dielectric substance is kept in an electric field E then electric field is used to create free charge as well as bound charge. Therefore surface integral of electric field be expressed in the form of →

$$\oint_s \vec{E}\cdot\vec{ds} = \frac{1}{\epsilon_0}(q+q') \text{ ——— } (1)$$

Free charges is associated with free charge current density and bound charge is associated with bound charge density

$$\left.\begin{aligned} q &= \int_V \rho\, dv \\ q' &= \int_V \rho'\, dv \end{aligned}\right\} \text{ ——— } (2)$$

$$\oint \vec{E}\cdot\vec{ds} = \frac{1}{\epsilon_0}\int_V (\rho+\rho')\, dV \text{ ——— } (A)$$

From eqn (1) and (2)
using Gauss divergence theorem →

$$\int_V \operatorname{div} E \cdot dV = \frac{1}{\epsilon_0}\int_V (\rho+\rho')\, dV$$

$$\rho' = -\operatorname{div} P$$

P is electric polarization

$$\int_V \mathrm{div}(\epsilon_0 E)\, dV = \int_V \rho\, dV \quad \int_V \mathrm{div}\, P\, dV$$

$$\int_V \mathrm{div}(\epsilon_0 E + P)\, dV = \int_V \rho\, dV$$

$$\int_V \mathrm{div}\, D\, dV = \int_V \rho\, dV \qquad [\because D = \epsilon_0 E + P]$$

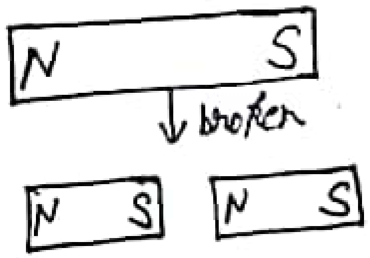
$$\Rightarrow \mathrm{div}\, \vec{D} = \rho$$

This is the differential form of Gauss law for electrostatics.

Maxwell second eq^n (Gauss law for magnetostatics) →
Acc. to this law div. of magnetic field is always equal to zero.
In mathematical form →

$$\mathrm{div}\, \vec{B} = 0$$

Proof → From the experiment it has been concluded that magnetic line of forces forms a closed loop or they may go to infinity. It means magnetic monopole does not exist at all.



$$\mathrm{div}\, \vec{B} = 0$$

Magnetic monopole does not exist.

Maxwell third eq^n → For time variable electric field curl of magnetic field intensity is equal to the sum of conduction current density and displacement current density.
In mathematical form this can be expressed as →

$$\text{curl}\, H = J + J_d$$

$$\text{curl}\, H = J + \frac{\partial D}{\partial t} \quad \text{——— (A)}$$

$$= J + \frac{\partial}{\partial t}(\epsilon_0 E)$$

$$= J + \epsilon_0 \frac{\partial E}{\partial t}$$

Proof → From Ampere circutal law line integral of mag. field intensity is equal to total current flowing in the loop.

$$\oint H \cdot dl = I \quad \text{——— (1)}$$

From the relation b/w current density and current

$$J = \frac{dI}{ds}.$$

$$dI = J\, ds$$

$$I = \int_S J\, ds \quad \text{——— (2)}$$

From eq^n (1) and (2)

$$\int H\, dl = \int J\, ds$$

using stoke's theorem →

$$\int_S \text{curl}\, H\, ds = \int_S J\, ds$$

$$\text{curl}\, H = J$$

Later on Maxwell found that this relation is inadequate. When time variable electric field is applied then extra term J_d is added in this relation.

Then modified relation becomes →

$$\text{curl}\, H = J + J_d$$

$$J_d = \frac{\partial D}{\partial t}$$

$$\text{curl}\, H = J + \frac{\partial D}{\partial t}$$

Maxwell fourth eq^n → This is based upon the Faraday law of electromagnetism. Acc. to which time variable magnetic field produces a electric field whose curl can be expressed as →

$$\text{curl}\, E = -\frac{\partial B}{\partial t}$$

Proof → According to Faraday law of electromagnetism time variable magnetic field produces a electric field and Value of induced emf →

$$\epsilon = -\frac{d\phi_B}{dt} \quad \text{———} (1)$$

ϕ_B = Magnetic flux

Induced E.M.F = - [time rate of change of mag. flux]

Here -ve sign signifies Lenz law according to which induced current oppose the region by which it is created.

From the definition of mag. flux →

$$d\phi_B = B \cdot ds$$

$$\phi_B = \int B \cdot ds \quad \text{———} (2)$$

using eqn (1) and (2)

$$\epsilon = -\frac{d}{dt}\int B \cdot ds \quad \text{———} (3)$$

$$dW = F \cdot dl$$

$$= qEdl$$

$$W = q\int Edl$$

If $q = 1$

$$\epsilon = \int E dl \quad \text{———} (4)$$

From (3) and (4)

$$\int E \cdot dl = -\frac{d}{dt}\int B ds \quad \text{———} (5)$$

By using Stoke's theorem →

$$\int \text{curl}\, E ds = -\int \frac{\partial B}{\partial t} ds$$

$$\int \left(\text{curl}\, E + \frac{\partial B}{\partial t}\right) ds = 0$$

$$\Rightarrow \quad \text{curl}\, E = -\frac{\partial B}{\partial t}$$

Integral form of Maxwell eq^n →

Maxwell I^{st} eq^n →

$$\operatorname{div} D = \rho$$

Multiplying from both the side by Volume element dv

$$\operatorname{div} D\, dv = \rho\, dv$$

Integrating on both the sides and then use Gauss theorem →

$$\int_S \vec{D} \cdot \vec{dS} = q$$

This is the integral form of I^{st} Maxwell eq^n which can be interpreted as surface integral of electric displacement vector is equal to free charge associated with it.

Maxwell II^{nd} eq^n →

$$\operatorname{div} B = 0 \text{ (Monopole does not exist)}$$

Multiplying on both the side by vol. element dv

$$\operatorname{div} B\, dv = 0$$

Integrating on both the side and using Gauss eq^N →

$$\int_V \operatorname{div} B\, dv = 0$$

$$\Rightarrow \int \vec{B} \cdot \vec{dS} = 0$$

This is the integral form of II^{nd} Maxwell eq^n which can be interpreted as surface integral of magnetic field vector across the total surface is equal to zero. It means magnetic line of forces formed a closed loop or in other words magnetic monopole does not exist.

Maxwell III^{rd} eq^n →

$$\operatorname{curl} H = J + J_d$$
$$= J + \frac{\partial D}{\partial t}$$

Taking the surface element ds on both the sides and then multiplying →

$\text{curl} H\, ds = J\, ds + \frac{\partial}{\partial t} D\, ds$

Integrating it

$$\int_S \text{curl} H\, ds = \int_S J\, ds + \int_S \frac{\partial}{\partial t} D\, ds$$

using the Stoke's theorem $\rightarrow$

$$\oint_L \vec{H} \cdot \vec{dl} = \int_S J\, ds + \frac{\partial}{\partial t} \int_S D\, ds$$

$$= I + I_D$$

Which can be interpreted as line integral of magnetic field intensity is equal to the sum of conduction current and displacement current. This is the integral form of Maxwell third eq^n.

Maxwell fourth eq^n $\rightarrow$

$$\text{curl} E = -\frac{\partial B}{\partial t}$$

Multiplying on both the sides by surface element ds

$$\text{curl} E\, ds = -\frac{\partial B}{\partial t} ds$$

and then taking the integration on both the sides using Stoke's theorem $\rightarrow$

$$\oint_L \vec{E} \cdot \vec{dl} = -\frac{d}{dt} \int B\, ds$$

$$\int_L \vec{E} \cdot dl = -\frac{d}{dt} \phi_B$$

Which can be interpreted as line integral of electric field is equal to -ve rate of change of magnetic flux. This is the integral form of Maxwell fourth eq^n.

Q1. Using Maxwell's equation, $\mathrm{div}E = \frac{\rho}{\epsilon_0}$, derive Coulomb's law of electrostatics.

Sol^n → Given, $\mathrm{div}E = \frac{\rho}{\epsilon_0}$ ——— (1)

Integrating the given relation (1) over volume V surrounding by the surface S, we get

$$\int_V \mathrm{div}\vec{E}\,dv = \int_V \frac{\rho}{\epsilon_0}\,dv \quad \text{———} (2)$$

From Gauss-divergence theorem, the volume integral is reduced to surface integral as,

$$\int_V \mathrm{div}E\,dV = \int_S E\cdot ds$$

$$\int_S E\cdot ds = \int_S \frac{\rho}{\epsilon_0}\,dv = \frac{q}{\epsilon_0}$$

For spherical surface of radius r around the charge q is

$$\int_S E\cdot ds = E(4\pi r^2)$$

$$E(4\pi r^2) = \frac{q}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

The force on charge q_0 in the electric field of charge q is

$$F = q_0 E = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r^2}$$

This is the required expression for Coulomb's law.

Q2 Using Maxwell's eq^n, $\mathrm{curl}B = \mu_0\left(J + \frac{\partial D}{\partial t}\right)$, prove that $\mathrm{div}\,D = \rho$

Sol^n → According to the given problem, $\mathrm{curl}B = \mu_0\left(J + \frac{\partial D}{\partial t}\right)$

Taking divergence of both sides we get

$$\mathrm{div}(\mathrm{curl}B) = \mu_0\,\mathrm{div}\left(J + \frac{\partial D}{\partial t}\right)$$

But div curl B = 0

$\therefore \text{div}\left(J+\frac{\partial D}{\partial t}\right)=0$

$\text{div}\, J+\frac{\partial}{\partial t}\text{div}\, D=0$

From eq of continuity, $\text{div}\, J=-\frac{\partial \rho}{\partial t}$

$\frac{\partial}{\partial t}(\text{div}\, D)=\frac{\partial \rho}{\partial t}$

$\text{div}\, D=\rho$

---

* wave impedance

$\Rightarrow$ $\mu_0 \omega \vec{H} = (\vec{k} \times \vec{E})$

$\mu_0 \omega \vec{H} = k(\hat{\eta} \times \vec{E})$ $\qquad \vec{k} = k\hat{\eta}$

$\mu_0 \omega |\vec{H}| = k\,|(\hat{\eta} \times \vec{E})|$

$\mu_0 \omega H = kE$

$H = \frac{k}{\mu_0 \omega} E$

$H = \frac{1}{\mu_0 \left(\frac{\omega}{k}\right) = c} \times E$

$H = \frac{1}{\mu_0 c} E$

$\Rightarrow$ $\frac{E}{H} = \mu_0 c$ $\qquad \because c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

$\frac{E}{H} = \mu_0 \times \frac{1}{\sqrt{\mu_0}\sqrt{\epsilon_0}} = \frac{\sqrt{\mu_0}}{\sqrt{\epsilon_0}}$

$\Rightarrow$ $\boxed{Z = \frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 376.7\,\Omega}$

e.m velocity = velocity of light

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Q1 Write the Maxwell's eqⁿ in integral and differential forms. Explain the physical significance of each eqⁿ. (2003, 2005)

Q2 Using Maxwell's equation, $\operatorname{div} E = \frac{\rho}{\epsilon_0}$, derive Coulomb's law of electrostatics.

→ $\vec{E}$, $\vec{H}$ & $\vec{k}$ are mutually perpendicular:-

$$\Rightarrow \quad \vec{\nabla} \longrightarrow ik$$

$$\vec{\nabla} \times \vec{E} = i(\vec{k} \times \vec{E})$$

$$-\mu_0 \frac{\partial \vec{H}}{\partial t} = i(\vec{k} \times \vec{E})$$

$$-\mu_0 \frac{\partial (H_0 e^{i(\vec{k}\cdot\vec{x} - \omega t)})}{\partial t} = i(\vec{k} \times \vec{E})$$

$$+\mu_0 \underbrace{H_0 e^{i(\vec{k}\cdot\vec{x} - \omega t)}}_{H} \cdot i\omega = i(\vec{k} \times \vec{E})$$

$$\boxed{\mu_0 H \omega = (\vec{k} \times \vec{E})}$$

$\vec{H}$, $\vec{k}$ & $\vec{E}$ are mutually perpendicular

) Poynting vector $\vec{S}$ = rate of flow of energy per unit area.

$$\boxed{\vec{S} = (\vec{E} \times \vec{H})} \quad \text{watt}/m^2$$

- Poynting vector and Poynting theorem

Imp ⇐ previous page → imp. note

Poynting theorem

The propagation of electromagnetic waves results in the transportation of energy from one place to other.

Maxwell's third and fourth equations are

$$\text{curl } E = -\frac{\partial B}{\partial t} \quad -1$$

and,
$$\text{curl } H = J + \frac{\partial D}{\partial t} \quad -2$$

Taking scalar product of equation(1) with H and equation (2) with E, we have

$$H.\text{curl } E = -H.\frac{\partial B}{\partial t} \quad -3$$

and
$$E.\text{curl } H = E.J + E.\frac{\partial D}{\partial t} \quad -4$$

Substract eq.(4) from eq.(3), we have

$$H.\text{curl } E - E.\text{curl } H = -H.\frac{\partial B}{\partial t} - E.\frac{\partial D}{\partial t} - E.J \quad -5$$

$$H.\text{curl } E - E.\text{curl } H = -\left(H.\frac{\partial B}{\partial t} + E.\frac{\partial D}{\partial t}\right) - E.J$$

using vector identity

$$\text{div}(E \times H) = H.\text{ Curl } E - E.\text{ curl } H$$

(17)

$$\operatorname{div}(E \times H) = -\left(H \cdot \frac{\partial B}{\partial t} + E \cdot \frac{\partial D}{\partial t}\right) - E \cdot J \qquad -6$$

using the relation $B = \mu H$; $D = \varepsilon E$ for a linear medium,

$$\operatorname{div}(E \times H) = -\left(\vec{H} \cdot \frac{\partial(\mu H)}{\partial t} + E \cdot \frac{\partial(\varepsilon E)}{\partial t}\right) - E \cdot J$$

$$= -\left(\mu \frac{\partial}{\partial t}(\vec{H} \cdot \vec{H}) + \varepsilon \frac{\partial}{\partial t}(\vec{E} \cdot \vec{E}) - \vec{E} \cdot \vec{J}\right.$$

$$= -\left\{\frac{1}{2}\mu \frac{\partial (H)^2}{\partial t} + \frac{1}{2}\varepsilon \frac{\partial (E)^2}{\partial t}\right\} - E \cdot J$$

$$= -\left\{\frac{1}{2}\frac{\partial (H \mu H)}{\partial t} + \frac{1}{2}\frac{\partial (E \cdot \varepsilon E)}{\partial t}\right\} - EJ$$

$$\cancel{= -\left\{\frac{1}{2}\frac{\partial (H \cdot B)}{\partial t} + \frac{1}{2}\frac{\partial}{\partial t}(E \cdot D)\right\} - EJ}$$

$$= -\left\{\frac{1}{2}\frac{\partial (H \cdot B)}{\partial t} + \frac{1}{2}\frac{\partial}{\partial t}(E \cdot D)\right\} - EJ$$

$$\operatorname{div}(E \times H) = -\frac{\partial}{\partial t}\left[\frac{1}{2}(E \cdot D + H \cdot B)\right\} - E \cdot J \qquad -7$$

Integrating equation (7) over a volume v bounded by a surface s, we have

$$\int_v \operatorname{div}(E \times H)\, dv = -\int_v \left\{\frac{\partial}{\partial t} \cdot \frac{1}{2}(E \cdot D + H \cdot B)\right\} dv - \int_v E J \, dv$$

$$\oint_s (E\times H)\,ds = -\frac{\partial}{\partial t}\int_v \frac{1}{2}(E\cdot D + H\cdot B)\,dv - \int_v J\cdot E\,dv$$

$$-\int_v J\cdot E\,dv = \frac{\partial}{\partial t}\int_v \frac{1}{2}(E\cdot D + H\cdot B)\,dv + \oint_s (E\times H)\,d$$

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The terms in equation (8) represent

(1) $-\int_v J.E\,dv$; The rate of transfer of energy into the electromagnetic field due to the motion of charge.

(2) $\frac{d}{dt}\int_v \frac{1}{2}(E.D + H.B)\,dv = U_e + U_m = U_{Total}$ and represents the rate of electromagnetic energy stored.

(3) The term $\int_s (E\times H)\,ds$ represents the amount of energy crossing per second through the closed surface.

And $(\vec{E}\times\vec{H}) = \vec{S}$ is known as Poynting vector. $\vec{S} = \vec{E}\times\vec{H}$ is the energy flowing through unit area and unit time and is known as the Poynting vector.

Electromagnetic waves in Fre[e space]

For free space

$\rho = 0; J = 0; \sigma = 0; \mu = 1; \varepsilon = 1; D = \varepsilon_0 E; B = \mu_0 H.$

Maxwell's equation for free space will be

$$\text{div} D = 0 \quad -1$$

$$\text{div} B = 0 \quad -2$$

$$\text{curl} E = -\frac{\partial B}{\partial t} \quad -3$$

$$\text{curl} H = \frac{\partial D}{\partial t} \quad -4$$

Take curl of equation 3, we have

$$\text{curl curl} E = -\frac{\partial}{\partial t}(\text{curl} B)$$

$$= -\frac{\partial}{\partial t}(\text{curl} \mu_0 H)$$

$$\text{curl curl} E = -\mu_0 \frac{\partial}{\partial t}(\text{curl} H) \quad -5$$

Using equation(4) in equation(5)

$$\text{curl curl} E = -\mu_0 \frac{\partial}{\partial t}\left(\frac{\partial D}{\partial t}\right) = -\mu_0 \frac{\partial}{\partial t}\left(\frac{\partial \varepsilon_0 E}{\partial t}\right)$$

$$= -\mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \quad -6$$

Using the vector identity

$$\text{curl curl} E = \text{grad div} E - \nabla^2 E$$

the equation (6) becomes

$$\text{grad div } E - \nabla^2 E = -\mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \quad -7$$

using equation (1), div D = 0 or div E = 0
we have grad div E = 0, equation (7) is

$$-\nabla^2 E = -\mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\boxed{\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}} \quad -8$$

Similarly

$$\boxed{\nabla^2 H = \mu_0 \epsilon_0 \frac{\partial^2 H}{\partial t^2}} \quad -9$$

Equation (8) and (9) are known as wave equation for E and H in free space.

The general wave equation is

$$\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \quad -10$$

Compare (8), (9) and (10),
the field vectors E and H are Propagated in free space with speed

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \sqrt{\frac{4\pi}{\mu_0 \, 4\pi \epsilon_0}} = \sqrt{\frac{4\pi \times 9 \times 10^{9}}{4\pi \times 10^{-7}}} = 3 \times 10$$

$$\boxed{v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c}$$

i.e Em. wa[ve]
with spee[d]
free space

Numericals

Q. Assuming that all the energy from a 1000 watt lamp is radiated uniformly, calculate the average values of the intensities of electric and magnetic fields of radiation at a distance of 2m from the lamp.

(UPTU, 2008, 2009, 2010)

Sol.

Here Energy of the lamp = 1000 watts

= 1000 Joules/sec

Area illuminated $= 4\pi r^2 = 4\pi (2)^2$

$= 16\pi$ meter^2

Hence, energy radiated per unit area per second

$$= \frac{1000}{16\pi}$$

From Poyntin theorem,

$$|S| = |E \times H| = EH = \frac{1000}{16\pi}$$

and $$\frac{E}{H} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.72 \qquad -(2)$$

Multiply eq. (1) and (2), we have

$$E^2 = \frac{1000}{16\pi} \times 376.72$$

$$E = 48.87 \text{ Volt/m}$$

Q. If the earth receives 2 cal min^{-1} cm^{-2} solar energy, what are the amplitudes of electric and magnetic fields of radiation?

(UPTU 2002, 2007)

Sol.

Poynting vector

$$|S| = |E \times H| = EH \sin 90 = EH$$

$$(E \perp H)$$

The energy received by earth surface is

$$|S| = 2 \text{ cal } \cancel{\ } \text{ min}^{-1} \text{ cm}^{-2}$$

$$= \frac{2 \times 4.2 \times 10^4}{60} \text{ J m}^{-2} \text{ sec}^{-1}$$

$$= 1400$$

So $EH = 1400$ —(1)

And $\frac{E}{H} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}} = 376.72$ ohms

—(2)

Multiplying equation (1) by (2), we have

$$E^2 = 1400 \times 376.72 = 5.274 \times 10^5$$

$$E = 726.2 \text{ volt/m}$$ —(3)

Using (E) from ~~equation~~ equation (3) in (1), we have

$$726.2\, H = 1400 \quad \text{or} \quad H = \frac{1400}{726.2}$$

$$H = 1.928 \text{ amp-turn/m}$$

are, the amplitudes of electric and magnetic fields of radiation are

$$E_0 = E\sqrt{2} = 726.2\sqrt{2} = 1026.8 \text{ volt/m}$$

and $$H_0 = H\sqrt{2} = 1.928\sqrt{2} = 2.726 \text{ amp-turn/m}$$

 641, 628

E.M. wave characteristics in free space

1. They are transverse waves consisting of variations of electric and magnetic field vectors ($\vec{E}$ & $\vec{B}$)
2. $\vec{E}$ and $\vec{B}$ are mutually perpendicular in the e.m. waves and are also perpendicular to the direction of propagation of wave.
3. magnitude of $\vec{E}$ and $\vec{B}$ (or $\vec{H}$) fields in an e.m. waves are proportional at any point. In free space we have

$$\frac{|\vec{E}|}{|\vec{B}|} = c = 2.9979 \times 10^{8}\ m/s \simeq 3\times 10^{8}\ m/s$$ (the speed of light in free space)

4. In free space, $\vec{E}$ & $\vec{B}$ (or $\vec{H}$) are in phase ie they become maximum and zero simultaneously (together).
5. The ($\vec{E}$, $\vec{B}$ & $\vec{k}$) form a Right handed coordinate system. where $\vec{k}$ is wave propagation vector.
6. The ratio of $|\vec{E}|$ and $|\vec{H}|$ ($=\frac{|\vec{B}|}{\mu_0}$, in free space) has dimensions of electrical impedance/resistance and is called wave impedance of free space & denoted by Z_0.

$$Z_0 = \frac{|E|}{|H|} = \sqrt{\frac{\mu_0}{\epsilon_0}} = \underline{\underline{376.6\ \Omega}}$$

7. For an e.m. wave in free space, energy is distributed equally between electric and magnetic fields.

electrostatic energy density, $u_e = \frac{\epsilon_0 E^2}{2}$

and magnetostatic energy density, $u_m = \frac{\mu_0 H^2}{2}$ $\left(= \frac{B^2}{2\mu_0}\right)$

$$\therefore \frac{u_e}{u_m} = \frac{\epsilon_0 E^2/2}{\mu_0 H^2/2} = \frac{\epsilon_0}{\mu_0}\frac{E^2}{H^2} = \frac{\epsilon_0}{\mu_0} Z_0^2 = \frac{\epsilon_0}{\mu_0}\frac{\mu_0}{\epsilon_0} = 1$$

$$\Rightarrow \boxed{u_e = u_m}$$ for e.m. wave in free space

Energy density in e.m. waves

If $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields (they are functions of position $\vec{x}$ and time t) in an e.m. wave then the Total energy present in a volume of space in which e.m. wave is propagating is given by:

Energy, $\boxed{U_{em} = \frac{1}{2}\int\left(\epsilon_0 E^2 + \frac{B^2}{\mu_0}\right) d\tau}$ (E & B are rms values of $\vec{E}$ & $\vec{B}$)

where $d\tau = dx\,dy\,dz$ is the volume element.

The energy in electric field is given by

$$\boxed{U_E = \frac{1}{2}\epsilon_0 E^2}$$

while energy in the magnetic field is given by

$\boxed{U_m = \frac{1}{2}\frac{B^2}{\mu_0}}$ since $\vec{B} = \mu_0 \vec{H}$ in free space

∴ in Terms of $\vec{H}$, $\boxed{U_m = \frac{1}{2}\mu_0 H^2}$

Total energy of e.m wave, $\boxed{U_{em} = U_E + U_m}$

Ratio of electric & magnetic energies:

$$\frac{U_E}{U_m} = \frac{\frac{1}{2}\epsilon_0 E^2}{\frac{1}{2}B^2/\mu_0} = \frac{\frac{1}{2}\epsilon_0 E^2}{\frac{1}{2}\mu_0 H^2} = \frac{\epsilon_0}{\mu_0}\frac{E^2}{H^2} = \frac{\epsilon_0 \mu_0}{\mu_0 \epsilon_0} = 1$$

since $\frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_0}}$ = wave impedance of free space (Z_0)

$\Rightarrow \boxed{U_E = U_m}$ Energy of an e.m. wave is divided equally between electric and magnetic fields.

∴ we can write $U_{em} = 2 \times \frac{1}{2}\epsilon_0 E^2$ or $\boxed{U_{em} = \epsilon_0 E^2}$

The e.m. waves carry mass (relativistic) given by Einsteins famous equation $E = mc^2$

$$\therefore \boxed{m = U/c^2} \quad \text{or} \quad \boxed{m = \frac{\epsilon_0 E^2}{c^2} = \frac{\mu_0 H^2}{c^2}}$$

Since $\boxed{\frac{|\vec{E}|}{|\vec{B}|} = c} \Rightarrow \boxed{m = \epsilon_0 B^2}$ using

$$\boxed{\begin{array}{l} \vec{k}\times\vec{E} = \omega\vec{B} \\ \left(\frac{k}{\omega}\right)\hat{k}\times\vec{E} = \vec{B} \\ \hat{k}\times\frac{\vec{E}}{\vec{B}} = \frac{\omega}{k} = c \\ \hat{k}\perp\vec{E} \Rightarrow |\hat{k}\times\vec{E}| = |\vec{E}| = E \end{array}}$$

The momentum carried by e.m. waves is given by:

$$\vec{p} = \epsilon_0\int(\vec{E}\times\vec{B})\,d\tau = \frac{1}{c^2}\int(\vec{E}\times\vec{H})\,d\tau = \frac{1}{c^2}\int\vec{S}\,d\tau$$

where we have used $\mu_0\epsilon_0 = \frac{1}{c^2}$ and $d\tau = dx\,dy\,dz$

and $\vec{S} = \vec{E}\times\vec{H}$ is the poynting vector.

The power per unit area transported by e.m. wave in its direction of propagation is represented by poynting vector. (Power is energy/time).

A charge q radiates e.m. waves and the power carried by these waves is related to the acceleration of the charge (a uniformly moving charge does not radiates energy) as given by Larmor Power formula

$$\boxed{P = \frac{\mu_0}{6\pi}\frac{q^2 a^2}{c}}$$

Time average of energy density, $\langle U_{em}\rangle = \langle \epsilon_0 E^2\rangle$

since $E = E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} = E_0\left[\cos(\vec{k}\cdot\vec{r}-\omega t) + i\sin(\vec{k}\cdot\vec{r}-\omega t)\right]$

$$\therefore U_{Em} = \langle\epsilon_0 E^2\rangle_{Real} = \epsilon_0 E_0^2\langle\cos^2(\vec{k}\cdot\vec{r}-\omega t)\rangle = \epsilon_0 E_0^2\left(\tfrac{1}{2}\right)$$

$$\Rightarrow \boxed{U_{Em} = \epsilon_0 E_{rms}^2}$$

Poynting vector has an average value which can be found :-

$$\vec{S} = \vec{E}\times\vec{H} = \frac{\vec{E}\times(\hat{k}\times\vec{E})}{\mu_0 c}$$ since $\vec{H} = \frac{1}{\mu_0\omega}(\vec{k}\times\vec{E})$

or $\vec{H} = \frac{1}{\mu_0 c}(\hat{k}\times\vec{E})$.

$$\Rightarrow \vec{S} = \frac{1}{\mu_0 c}\left[\hat{k}(\vec{E}\cdot\vec{E}) - \vec{E}(\vec{E}\cdot\hat{k})\right]$$ where $\hat{k}$ is unit vector along $\vec{k}$.

(using rule for triple cross product $\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$)

but $\vec{E}\cdot\hat{k} = 0$ since $\vec{E}\perp\hat{k}$

and $\vec{E}\cdot\vec{E} = E^2$

$$\therefore \boxed{\vec{S} = \frac{E^2}{\mu_0 c}\hat{k}}$$

Now time average of poynting vector

$$\langle\vec{S}\rangle = \frac{1}{\mu_0 c}\langle E^2\rangle\hat{k} = \frac{1}{\mu_0 c}E_0^2\left\langle\left(e^{i(\vec{k}\cdot\vec{r}-\omega t)}\right)^2\right\rangle_{Real}\hat{k}$$

$$\left(\text{or } \langle\vec{S}\rangle = \sqrt{\frac{\mu_0}{\epsilon_0}}E_0^2\langle\cos^2(\vec{k}\cdot\vec{r}-\omega t)\rangle \quad \text{since } c = \frac{1}{\sqrt{\mu_0\epsilon_0}}\right)$$

$$\boxed{\langle\vec{S}\rangle = \frac{1}{\mu_0 c}\frac{E_0^2}{2}\hat{k} = \frac{E_{rms}^2\hat{k}}{\mu_0 c}}$$ Energy flow is along direction of wave propagation $(\vec{k})$.

$$\therefore \text{Ratio of } \frac{\langle S\rangle}{\langle U_{Em}\rangle} = \frac{E_{rms}^2/\mu_0 c}{\epsilon_0 E_{rms}^2} = \frac{1}{\epsilon_0\mu_0 c} = \frac{c^2}{c} = c$$

or $$\boxed{\langle S\rangle = \langle U_{Em}\rangle c\,\hat{k}}$$

Energy flux = energy density × speed of light

In other words, an e.m. wave carries energy density in it at the speed of light

— × —

Propagation of e.m. wave in non-conducting medium

In nonconducting medium, $\sigma = 0$ and $\rho = 0$

The Maxwell's equations for a nonconductive medium are

$$\vec{\nabla} \cdot \vec{E} = 0 \quad \text{—— (1)}$$

$$\vec{\nabla} \cdot \vec{H} = 0 \quad \text{—— (2)}$$

$$\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad \text{—— (3)}$$

$$\vec{\nabla} \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t} \quad \text{—— (4)}$$

Taking curl of equn.(3), we get

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\mu \vec{\nabla} \times \frac{\partial \vec{H}}{\partial t} = -\mu \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{H})$$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{H}) \quad \text{—— (5)}$$

Substitute the value of $\vec{\nabla} \cdot \vec{E}$ and $\vec{\nabla} \times \vec{H}$ in equn.(5)

$$-\nabla^2 \vec{E} = -\mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\boxed{\nabla^2 \vec{E} = \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}} \quad \text{—— (6)}$$

Similarly we can derive, $\nabla^2 \vec{H} = \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2}$ —— (7)

Compare equn.(6) & (7) with general wave equn. of the form

$$\nabla^2 u = \frac{1}{v^2} \frac{\partial^2 u}{\partial t^2}$$

we get

$$v = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}}$$

$$\boxed{v = \frac{c}{\sqrt{\mu_r \epsilon_r}}}$$

L30

Propagation of E m Waves in Conducting Medium

Maxwell's equations for conducting medium

$$\vec{\nabla}\cdot\vec{D} = 0 \quad \text{---(1)} \qquad (\because \rho = 0 \text{ for a conductor})$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \text{---(2)}$$

$$\vec{\nabla}\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t} \quad \text{---(3)}$$

$$\vec{\nabla}\times\vec{H} = \sigma\vec{E} + \epsilon\frac{\partial\vec{E}}{\partial t} \quad \text{---(4)}$$

Taking curl of both sides of equn.(3)

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = -\mu\,\vec{\nabla}\times\frac{\partial\vec{H}}{\partial t}$$

$$\Rightarrow \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E} = -\mu\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{H})$$

$$\Rightarrow \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E} = -\mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2} - \mu\sigma\frac{\partial\vec{E}}{\partial t} \qquad \left[\because \text{from equn.(1)}\ \vec{\nabla}\cdot\vec{E} = 0\right]$$

$$\Rightarrow -\nabla^2\vec{E} = -\mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2} - \mu\sigma\frac{\partial\vec{E}}{\partial t}$$

$$\Rightarrow \boxed{\nabla^2\vec{E} - \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2} - \mu\sigma\frac{\partial\vec{E}}{\partial t} = 0} \quad \text{---(5)}$$

Similarly we can derive

$$\nabla^2\vec{H} - \mu\epsilon\frac{\partial^2\vec{H}}{\partial t^2} - \mu\sigma\frac{\partial\vec{H}}{\partial t} = 0 \quad \text{---(6)}$$

Now equn.(5) can be written as

$$\frac{\partial^2\vec{E}}{\partial x^2} + \frac{\partial^2\vec{E}}{\partial y^2} + \frac{\partial^2\vec{E}}{\partial z^2} = \mu\epsilon\frac{\partial^2 E}{\partial t^2} + \mu\sigma\frac{\partial E}{\partial t} \quad \text{---(7)}$$

Let us consider a plane em wave propagating in z dir. with vector $\vec{E}$ in x direction and vector $\vec{H}$ in Y direction.

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

and $\vec{E} = E_x\hat{a}_x$ and $\vec{H} = H_y\hat{a}_y$

Equn.(7) can be written as

$$\frac{\partial^2 E_x}{\partial z^2} = \mu\epsilon\frac{\partial^2 E_x}{\partial t^2} + \mu\sigma\frac{\partial E_x}{\partial t} \quad \text{---(8)}$$

If electric field E_x varies harmonically with time, th

$$E_x = E_0 e^{j\omega t}$$

$$\frac{\partial E_x}{\partial t} = E_0(j\omega)e^{j\omega t} = j\omega E_x$$

Substituting these values in equn. (8), we get

$$\frac{\partial^2 E_x}{\partial z^2} = -\mu\epsilon\omega^2 E_x + j\omega\mu\sigma E_x$$

or, $$\frac{\partial^2 E_x}{\partial z^2} = (j\omega\mu\sigma - \omega^2\mu\epsilon)E_x$$

Let us put γ (propagation constant) as $\gamma^2 = (j\omega\mu\sigma - \omega^2\mu\epsilon)$ —

$$\frac{\partial^2 E_x}{\partial z^2} - \gamma^2 E_x = 0 \qquad \text{— (10)}$$

Solution of this equn. is of the form

$$E_x = E_m e^{-\gamma z} \qquad \text{— (11)}$$

where $\gamma^2 = j\omega\mu\sigma - \omega^2\mu\epsilon$

$\Rightarrow (\alpha + j\beta)^2 = j\omega\mu\sigma - \omega^2\mu\epsilon$

$\Rightarrow \alpha^2 - \beta^2 + 2j\alpha\beta = j\omega\mu\sigma - \omega^2\mu\epsilon$

Comparing real and imaginary parts, we get

$$\alpha^2 - \beta^2 = -\omega^2\mu\epsilon \qquad \text{— (12)}$$

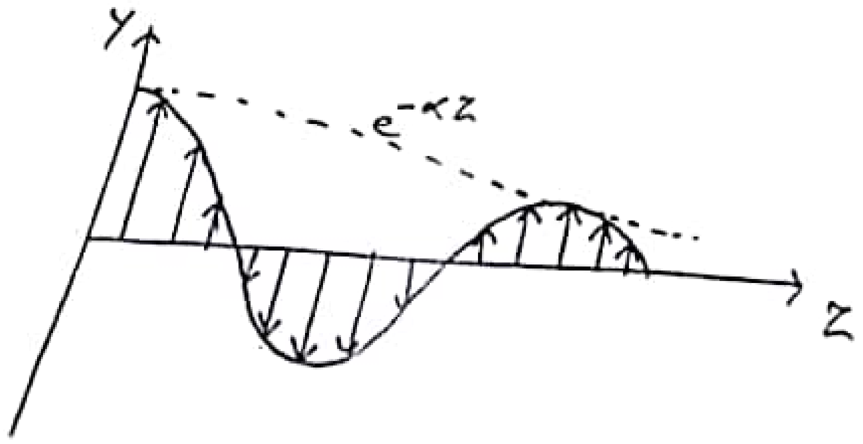
$$2\alpha\beta = \omega\mu\sigma \qquad \text{— (13)}$$

After solving equn. (12) & (13), we get

Attenuation constant $$\alpha = \omega\sqrt{\left(\frac{\mu\epsilon}{2}\right)\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]} \qquad \text{— (14)}$$

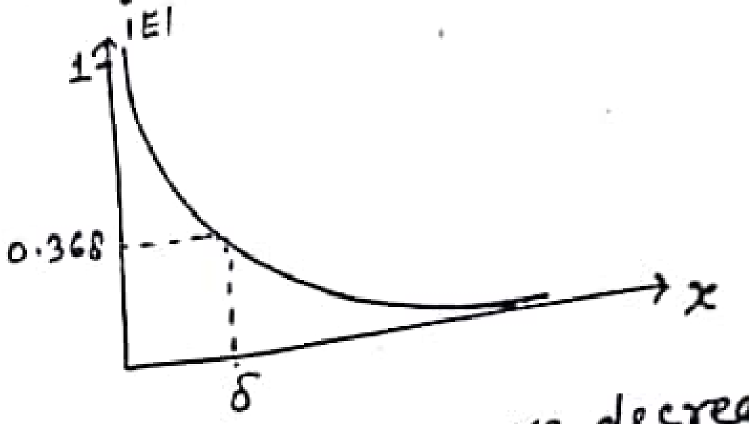
Phase constant $$\beta = \omega\sqrt{\left(\frac{\mu\epsilon}{2}\right)\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]} \qquad \text{— (15)}$$

The reduction of amplitude of a uniform plane wave as it travels through a conducting medium is shown in figure



Skin Depth (δ) — The skin depth is defined as ... which the strength of electric field associated with the e.m. waves reduces to $\left(\frac{1}{e}\right)$ times of its initial value.

fig shows variation of E with distance.



The amplitude of electric field strength of e.m. wave decreases by a factor $e^{-\alpha x}$ where α is attenuation constant.

The distance for which amplitude of e.m. wave decreases by a factor $\left(\frac{1}{e}\right)$ of its initial value is given by

$$\alpha x = 1$$

Now $x = \delta$ where δ is skin depth

$$\Rightarrow \quad \alpha \delta = 1$$

$$\delta = \frac{1}{\alpha}$$

So $\boxed{\text{skin depth} = \frac{1}{\text{Attenuation Constant}}}$

for good conductors —

skin depth $\boxed{\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega \mu \sigma}}}$

for good dielectrics —

skin depth $\boxed{\delta = \frac{1}{\alpha} = \frac{1}{\frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}}}$

Ques.- The permeability, permittivity and conductivity of alumimium are $\mu_r = 1$, $\epsilon_r = 1$ and $\sigma = 3.54 \times 10^7$ mho/m find the skin depth if the wave enter in aluminium with frequency of 71.56 MHz. (2008, 2009)

Solution- The skin depth of a good conductor is given by

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

Given that $\mu_r = 1$ i.e. $\mu = \mu_0 \mu_r = 4\pi \times 10^{-7}$

$\sigma = 3.54 \times 10^7$ mho/m

and $f = 71.56 \times 10^6$ Hz

$$\therefore \delta = \frac{1}{\sqrt{3.14 \times (71.56 \times 10^6) \times (4 \times 3.14 \times 10^{-7}) \times (3.54 \times 10^7)}}$$

$= 10 \times 10^{-6}$ m

$= 10\, \mu m$

Engineering Physics - II

Subject Code – RAS 201 (ODD)

ASSIGNMENT 3

1. Write down the Maxwell's equation in free space and show that the velocity of plane electromagnetic waves in free space is given by $c = 1/(\mu_0 \epsilon_0)^{1/2}$. (2007, 2008, 2010, 2013, 2015)

2. What is displacement current? How it will lead to modification of ampere circuital law. (2003, 2006, 2007, 2008, 2010, 2012, 2013, 2015)

3. Derive Maxwell's equation. Give physical significance of each. (2009, 2011)

4. Discuss Poynting theorem for flow of energy in electromagnetic field and define Poynting vector. (2008, 2012, 2015)

5. Write down the Maxwell's equation in conducting medium and using these equation derive wave equation for both electric and magnetic field. Also show that electric and magnetic field strength decrease exponentially with distance from the surface in to the conducting medium. (2008, 2010, 2012, 2014)

TUTORIAL 6

1. A bar magnet has a coercivity 5×10^3 amp/m. It is desired to demagnetize it by inserting it inside a solenoid 10 cm long and having 50 turns. What current should be sent through the solenoid? (2002)

2. The area of a B-H loop is 0.5 cm^2, whereas on X-axis 1 cm represents $H = 10^3$ A/m, and on Y-axis 1 cm represents B = 1T. If the volume of the specimen is $10^{-3} m^3$ and the frequency of a.c. is 50 Hz, calculate the hysteresis loss.

3. A parallel plate capacitor is being charged. Show that displacement current across an area in the region between the plates and parallel to it is equal to the conduction current in the connecting wires.

4. Obtain Coulomb's law of electrostatics using Maxwell's equation.

5. In a material for which σ = 5 S/m and ϵ_r = 1, the electric field intensity is $E = 250 \sin(10^{10}t)$ V/m. Find the conduction and displacement current densities and the frequency at which both have equal amplitude.

TUTORIAL 7

1. If the earth receives 2 cal/min-cm^2 solar energy, what are the amplitudes of electric and magr
fields of radiation. (2002, 2007, 20

2. Assuming that all the energy from a 1000 wall lamp is radiating uniformly, calculate the average val
of the intensities of electric and magnetic fields of radiation at a distance of 2 m from the lar
(2008. 2009, 2010)

3. If a plane electromagnetic wave in free space has magnitude of H 1 ampere/m .What is the magnitu
of E? (201

4. If the upper atmospheric layer of earth receives 1360 Wm^2 energy from the sun, what will be the pea
values of electric and magnetic fields at the layer? (2011

5. The relative permittivity of distilled water is 81. Calculate refractive index and velocity of light in i
(2000)