FORCE:

A Force is defined as an agent which produces or tends to produce, destroy or to destroy motion.

EFFECTS OF A FORCE:

A Force may produce the following effects on a body, on which it acts:

(1) It may change the motion of a body, i.e. if a body is at rest, the force may set in motion. And if the body is already in motion, the force may accelerate it.

(2) It may retard the motion of a body.

(3) It may retard the forces, already acting on a body, thus bringing it to rest or in equilibrium.

(4) It may give rise to the internal stresses in the body, on which it acts. (We shall study in Tru

CHARACTERSTICS OF A FORCE:

In order to determine the effects of a force, acting on a body, we must know the following characteristics of a force:

1. Magnitude of the force (i.e., 100N, 50N, 20 KN, etc.)
2. The line of action of the force. (i.e., along OX, OY, at 30° North of East etc.)
3. Nature of the Force (i.e, whether the force is push or pull). This is denoted by placing an arrow head on the line of action of the force.
4. The point at which (or through which) the force acts on the body.

PRINCIPLE OF PHYSICAL INDEPENDENCE OF FORCES:

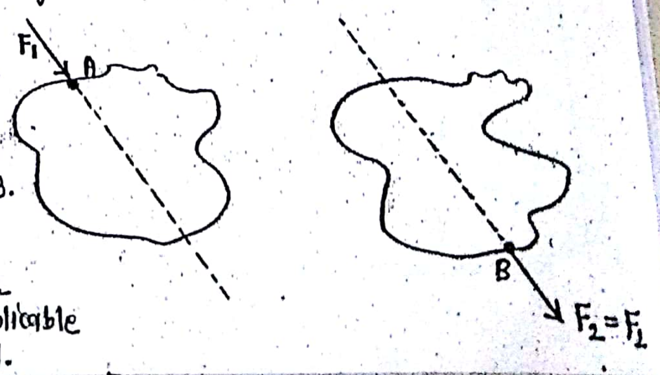
It states, "If a number of forces are simultaneously acting on a particle, then resultant of these forces will have the same effects as produced by all the forces."

PRINCIPLE OF TRANSMISSIBILITY OF FORCES:

"According to this law, the state of rest or motion of the rigid body is unaltered, if a force acting on the body is replaced by another force of the same magnitude and direction but acting anywhere on the body along the line of action of the replaced force.

Let F_1 be the Force acting on a rigid body at point A as shown in Fig(a).

According to the law of transmissibility of force, this force has the same effect on the state of body as the Force F_1 applied at point B.



Fig(a) Fig(b)

NOTE: In using law of transmissibility of force it should be carefully noted that, it is applicable only, if the body can be treated as rigid.

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PROOF:

The law of transmissibility of forces can be proved by using the law of superposition, which can be stated as " The action of a given system of forces on a rigid body is not changed by adding or subtracting another system of forces in equilibrium.

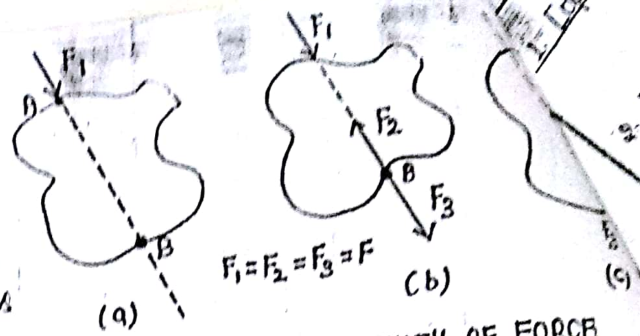


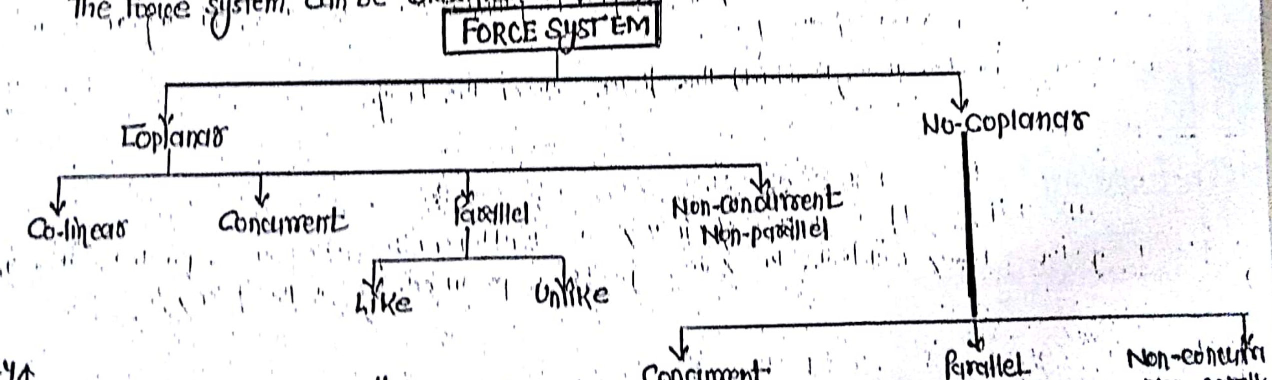
Fig: LAW OF TRANSMISSIBILITY OF FORCE

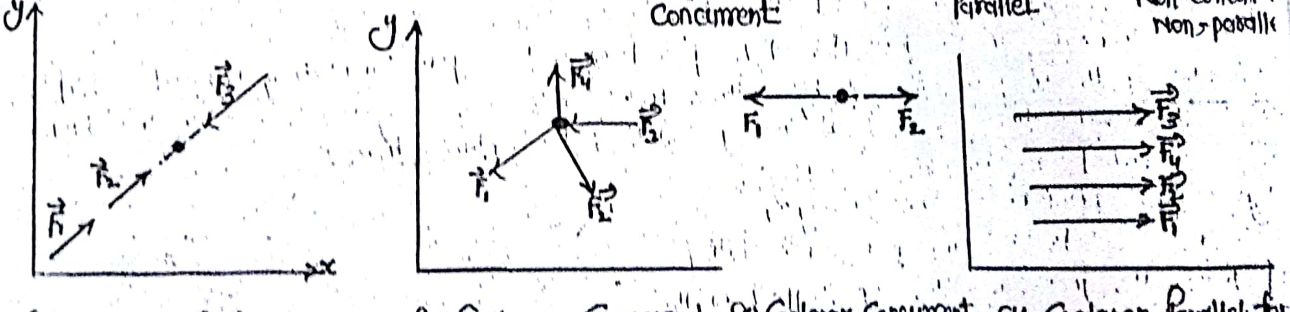
Consider the body shown in Fig(a). It is subjected to a force F_1 at A. B is another point on the line of action of the force. From the law of superposition it is obvious that, if two equ and opposite forces of magnitude F are applied at B along the line of action of the given for F (Ref. fig(b)) the effect of given force on the body is not altered. Force F_1 at A and oppo Force F_2 at B form a system of forces in equilibrium. If these two forces are subtracted from the system, the resulting system is as shown in fig(c). Looking at the system of forces in fig (a) we can conclude the law of transmissibility of forces is proved.

SYSTEM OF FORCES:

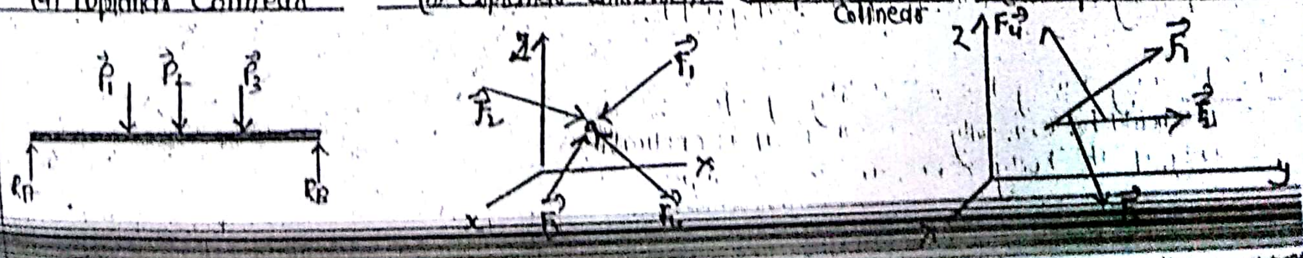
When several forces of different magnitude and direction act upon a body, they constitute a system of forces.

The force system can be classified as follows:

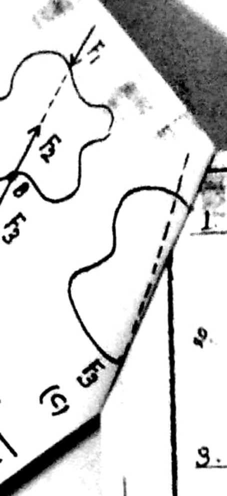




(a) Coplanar Colinear (b) Coplanar Concurrent (c) Coplanar Concurrent Colinear (d) Coplanar Parallel for



(e) Coplanar parallel forces (f) Non-coplanar concurrent forces (g) Non-coplanar non-concurrent



1. Coplanar Forces:
The Forces, whose lines of action lie on the same plane, are known as coplanar fo

2. Co-Linear Forces:
The forces whose lines of action lie on the same line, are known as colinear forces

3. CONCURRENT FORCES:
The forces, which meet at one point, are known as concurrent forces. The Concurrent forces may or may not colinear.

4. Coplanar Concurrent Forces:
The forces, which meet at one point and their lines of action also lie on the same plane, are known as Coplanar Concurrent forces.

5. Coplanar Non-concurrent forces:
The forces, which do not meet at one point, but their lines of action lie on the same plane, are known as coplanar non-concurrent forces.

6. Non-Coplanar-Nonconcurrent forces:
The forces, which (meet) do not meet at one point and their lines of action do not lie on the same plane, are called non-coplanar non-concurrent forces.

7. Non-Coplanar Concurrent forces:
The forces, which meet at one point, but their lines of action do not lie on the same plane, are known as non-coplanar concurrent forces.

RESULTANT FORCE:

If a number of forces, P, Q, R... etc. are acting simultaneously on a particle then it is possible to find out a single force which could replace them. i.e., which will produ the same effect as produced by all the given forces. This single force is called 'resultant forc and the given forces P, Q, R... etc are called Component forces.

COMPOSITION OF FORCES:

The process of finding out the resultant force, of a number of given forces, is calle Composition of forces or Compounding of forces.

METHOD FOR THE RESULTANT FORCE:

1. Analytical method
2. Method of resolution.

ANALYTICAL METHOD FOR RESULTANT FORCE:

[redacted]
2. Method of resolution.

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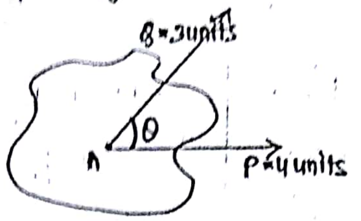
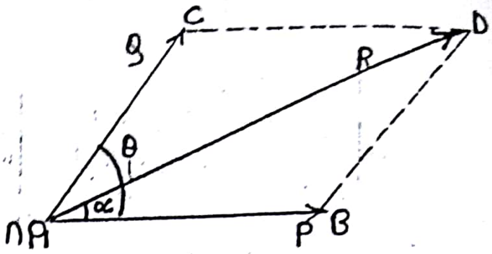
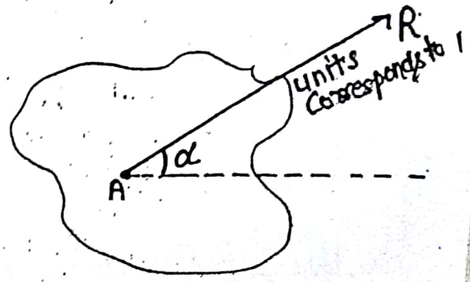
PARALLELOGRAM LAW OF FORCES:

It states " If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram; their resultant may be represented in magnitude and direction by the diagonal of the parallelogram, which passes through their point of intersection."





In Fig. the force $P = 4$ units and force $Q = 3$ units are acting on a body at a point A. Then to get the resultant of these forces parallelogram ABCD is constructed. such that $AB = 4$ units on linear scale and $AC = 3$ units. Then according to this law, the direction of AD represents the resultant in the direction and magnitude. Thus the resultant of the forces P and Q on the body is equal to units corresponding to AD in the direction α to P.



Mathematically resultant force:

$$\boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}}$$

and

$$\boxed{\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}}$$

Where: P and Q = Forces whose resultant is required to be found out.

θ = Angle between the forces P and Q, and

α = Angle which the resultant force makes with one of the forces (say P.)

NOTE: If the two forces are equal, i.e., when $P = Q = F$

$$R = \sqrt{F^2 + F^2 + 2F^2\cos\theta}$$

$$R = \sqrt{2F^2(1+\cos\theta)}$$

$$R = \sqrt{2F^2 \times 2\cos^2\left(\frac{\theta}{2}\right)} \quad \ldots\ldots \left[\because 1+\cos\theta = 2\cos^2\frac{\theta}{2}\right]$$

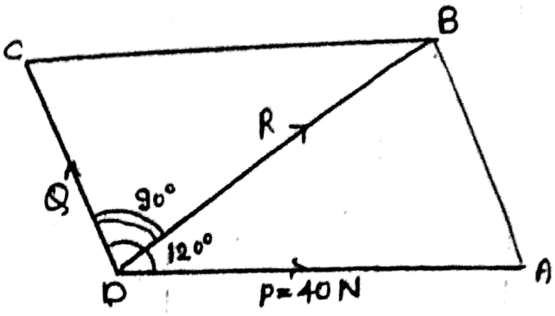
$$\boxed{R = 2F\cos\left(\frac{\theta}{2}\right)}$$

Q=1= Two forces of 100 N and 150 N are acting simultaneously at a point. what is the resultant of these two forces, if the angle between them is 45°?

Soln: $R = \sqrt{(100)^2 + 150^2 + 2 \times 100 \times 150 \times \cos 45^\circ}$

$\boxed{R = 232\ \text{N} \quad \text{Ans}}$

Q-2: Two forces acts at an angle of 120°. The bigger force is of 40N and the resultant perpendicular to the smaller one. Find the smaller Force.



Soln: Given: Angle between $\angle AOC = 120°$

Bigger Force $(P) = 40\,N$ and

angle between the resultant force and Q $(\angle BOC = 90°)$

Let Q = Smaller force in Newton

From the geometry of the figure, we find that

$\angle AOB$, $\alpha = 120° - 90° = 30°$

We know that; $\tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta}$

$$\tan 30° = \frac{Q \sin 120°}{40 + Q\cos 120°} =$$

$$0.577 = \frac{Q \times 0.866}{40 + Q(-0.5)}$$

$$\boxed{Q = 20N \quad \text{Ans}}$$

Q-3: Find the magnitude of the two forces, such that if they act at right angle, their resultant is $\sqrt{10}$ N. But if they act at 60°, their resultant is $\sqrt{13}$ N.

Soln: Given: Two Forces $= F_1$ and F_2

First of all Consider the two forces acting at right angles. we know that when the angle between the two given forces is 90°, then the resultant force (R)

$$\sqrt{10} = \sqrt{F_1^2 + F_2^2}$$

$$10 = F_1^2 + F_2^2 \quad \ldots\ldots (i)$$

Similarly, when the angle between the two forces is 60°, then the resultant force (R)

$$\sqrt{13} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 60°}$$

$$\therefore\ 13 = F_1^2 + F_2^2 + 2F_1F_2\cos 60°$$

$$13 = 10 + 2F_1F_2 \times (0.5)$$

$$F_1F_2 = 3 \quad \ldots\ldots \text{(substituting } F_1^2 + F_2^2 = 10)$$

we know that $(F_1 + F_2)^2 = F_1^2 + F_2^2 + 2F_1F_2$

$$= 10 + 2 \times 3 = 16$$

$$\therefore\ F_1 + F_2 = \sqrt{16} = 4 \quad \ldots\ldots (ii)$$

Similarly $(F_1 - F_2)^2 = F_1^2 + F_2^2 - 2F_1F_2$

$$= 10 - 6 = 4$$

$$\therefore\ F_1 - F_2 = 2 \quad \ldots (iii)$$

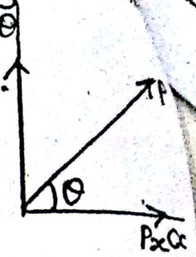
Solving equations (ii) and (iii)

$$F_2 = 1N$$

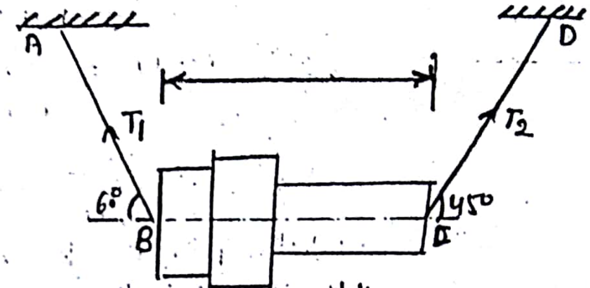
RESOLUTION OF A FORCE:

The process of splitting up the given force into a number of components without changing its effect on the body is called resolution of a force. A force, is generally resolve along two mutually perpendicular directions. Infact, the resolution of a force is the reverse action of the addition of the component vectors.

NOTE: In general, the forces are resolved in the vertical and horizontal directions.



Q-1: A machine component 1.5 m long and weight 1000N is supported by two rope AB and OD as shown in fig. Calculate the tensions T_1 and T_2 in the ropes AB and CD.



Solution: Given; Weight of component W = 1000N

Resolving the forces horizontally (i.e along BC)

$$T_1 \cos 60° = T_2 \cos 45°$$

$$T_1 = \frac{\cos 45°}{\cos 60°} T_2$$

$$T_1 = 1.414 T_2 \quad \ldots\ldots (i)$$

and Now resolving the forces vertically.

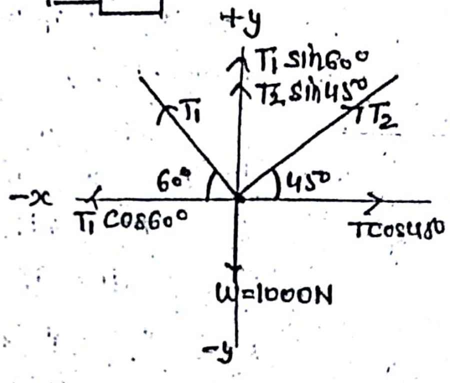
$$T_1 \sin 60° + T_2 \sin 45° = 1000$$

$$T_2 (1.414)\, 0.866 + T_2 \times 0.707 = 1000$$

$$1.93\, T_2 = 1000$$

$$\boxed{T_2 = 518.1\ N}$$

$$\boxed{T_1 = 732.6\ N}$$



METHOD OF RESOLUTION FOR THE RESULTANT FORCE:

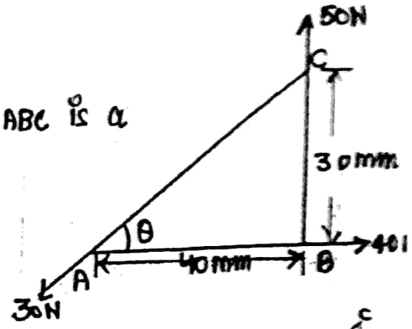
1. Resolve all the force horizontally and find the algebric sum of all the horizontal components (i.e., ΣH).
2. Resolve all the forces vertically and find the algebric sum of all the vertical components (i.e., ΣV)
3. The Resultant R of the given forces will be given by the equation.

$$\boxed{R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}}$$

4. The resultant force will be inclined at an angle θ, with the horizontal, such that.

$$\boxed{\tan\theta = \frac{\Sigma V}{\Sigma H}}$$

5- A triangle ABC has its side AB = 40 mm along positive x-axis and side BC = 30 mm along positive y-axis. Three forces of 40N, 50N, and 30 N act along the sides AB, BC and CA respectively. Determine magnitude of the resultant of such a system of forces

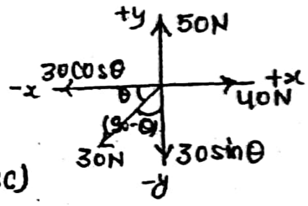


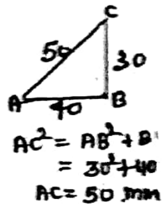
Soln: The system of given forces is shown in fig:

From the geometry of the figure, we find the triangle ABC is a right angled triangle, in which side AC = 50 mm.

Therefore; $\sin\theta = \frac{30}{50} = 0.6$

$\cos\theta = \frac{40}{50} = 0.8$





Resolving all the forces horizontally (i.e., along AB).

$\Sigma H = 40 - 30\cos\theta$

$\Sigma H = 40 - 30 \times 0.8 = 16\ N$

$\Sigma H = 16N$

and now resolving all the forces vertically (i.e. along BC)

$\Sigma V = 50 - 30\sin\theta$

$\Sigma V = 50 - 30 \times 0.6 = 32N$

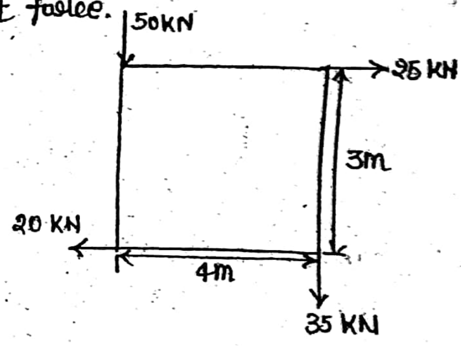
$\Sigma V = 32N$

We know that magnitude of the resultant force,

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(16)^2 + (32)^2}$

$\boxed{R = 35.8\ N}$ Ans

Q-6- A System of forces are acting at the corners of a rectangle block as shown in fig: Determine the magnitude and direction of the resultant force.



Soln:- Given; System of forces

Magnitude of the resultant force:

Resolving forces horizontally.

$\Sigma H = 25 - 20 = 5KN$

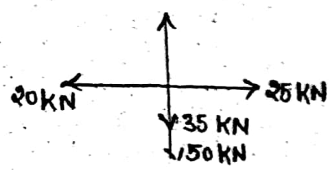
and now resolving the forces vertically,

$\Sigma V = (-50) + (-35) = -85\ KN$

∴ Magnitude of the resultant force

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(5)^2 + (-85)^2}$

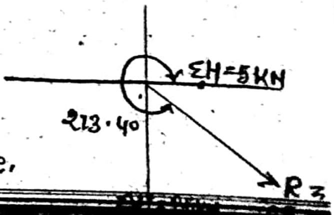
$\boxed{R = 85.15\ KN}$ Ans



Direction of the resultant force:

We know that, $\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{-85}{5} = -17$

$\boxed{\text{or } \theta = 86.6^\circ}$



Since ΣH is positive and ΣV in negative, therefore resultant lies between 270° and 360°. Thus actual angle of the resultant force,

$\theta = 213.4^\circ$; $\boxed{\theta = 213.4^\circ}$ Ans

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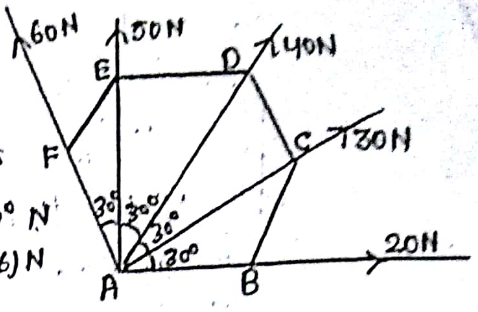
Q=7= The Forces 20N, 30N, 40N, 50N and 60N are acting at one of the angular points of a regular hexagon, towards the other five angular points taken in order. Find the magnitude and direction of the resultant force.



Solution: Resolving the forces horizontally (i.e along AB)

$\Sigma H = 20 + 30\cos 30° + 40\cos 60° + 50\cos 90° + 60\cos 120°$ N

$= 20 + (30\times 0.866) + (40\times 0.5) + (50\times 0) + (60\times(-0.5))$

$\Sigma H = 36.0$ N ....(i)

and now resolving the all forces vertically (i.e., at right angles to AB)

$\Sigma V = 20\sin 0° + 30\sin 30° + 40\sin 60° + 50\sin 90° + 60\sin 120°$ N

$=(20\times 0) + (30\times 0.5) + (40\times 0.866) + (50\times 1) + (60\times 0.866)$ N

$\Sigma V = 151.6$ N .....(ii)

We know that magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(36.0)^2 + (151.6)^2} = 155.8 \text{ N}$$

$$\boxed{R = 155.8\text{ N}}$$

Direction of the resultant force:

$$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{151.6}{36.0} = 4.211$$

$$\boxed{\theta = 76.6°}$$ Ans

Q=8= The following forces act at a point:

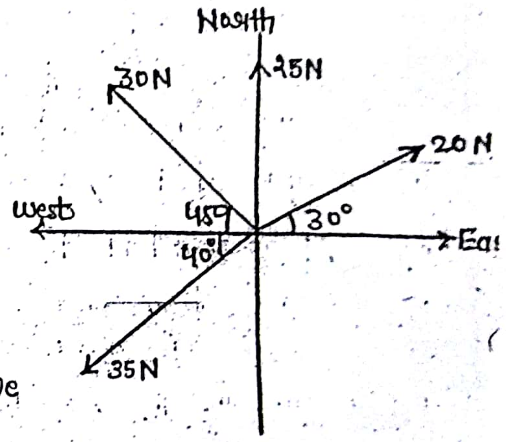
(i) 20 N inclined at 30° towards North of East,

(ii) 25 N towards North,

(iii) 30 N towards North west, and

(iv) 35 N inclined at 40° towards South of west.

Find the magnitude and direction of the resultant force.



Solution: Magnitude of the resultant Force:

Resolving all the forces horizontally, i.e along East - west line

$\Sigma H = 20\cos 30° - 30\cos 45° - 35\cos 40°$

$= (20\times 0.866) - (30\times 0.707) - 35(-0.766)$ N

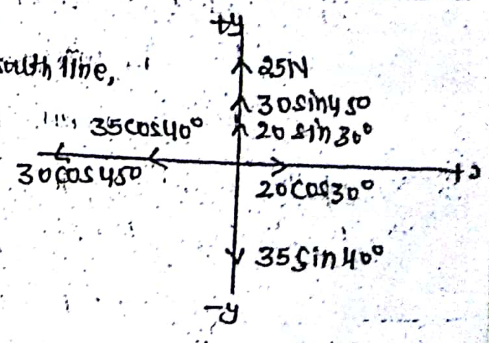
$\Sigma H = -30.7$ N ....(i)

and now resolving all the forces vertically i.e, along North-south line,

$\Sigma V = 20\sin 30° + 30\sin 45° + 25 - 35\sin 40°$

$= (20\times 0.5) + (30\times 0.707) + 25 - 35(+0.6428)$ N

$\Sigma V = 33.7$ N



We know that magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(-30.7)^2 + (33.7)^2}$$

$$\boxed{R = 45.6\text{ N}}$$

Direction of the resultant force:

$$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{33.7}{-30.7} = -1.098 \quad \text{or} \quad \theta = 47.7°$$

Since ΣH is negative and ΣV is positive, therefore resultant lies between 90° and 180°. The angle of the resultant, $\boxed{\theta = 180° - 47.7° = 132.3°}$ Ans

LAWS FOR THE RESULTANT FORCE:

The resultant force, of a given system of forces, may also be found out by the following laws:

1. Triangle law of forces
2. Polygon law of forces.

TRIANGLE LAW OF FORCES:

It states, "If two forces acting simultaneously on a particle, be represented in magnitude and direction by the two sides of a triangle taken in order; their resultant may be represented in magnitude and direction by the third side of the triangle, taken in opposite order."

POLYGON LAW OF FORCES:

It is an extension of Triangle law of forces for more than two forces, which states "If a number of forces acting simultaneously on a particle, be represented in magnitude and direction, by the sides of a polygon taken in order; then the resultant of all these forces may be represented, in magnitude and direction, by the closing side of the polygon, taken in opposite order.

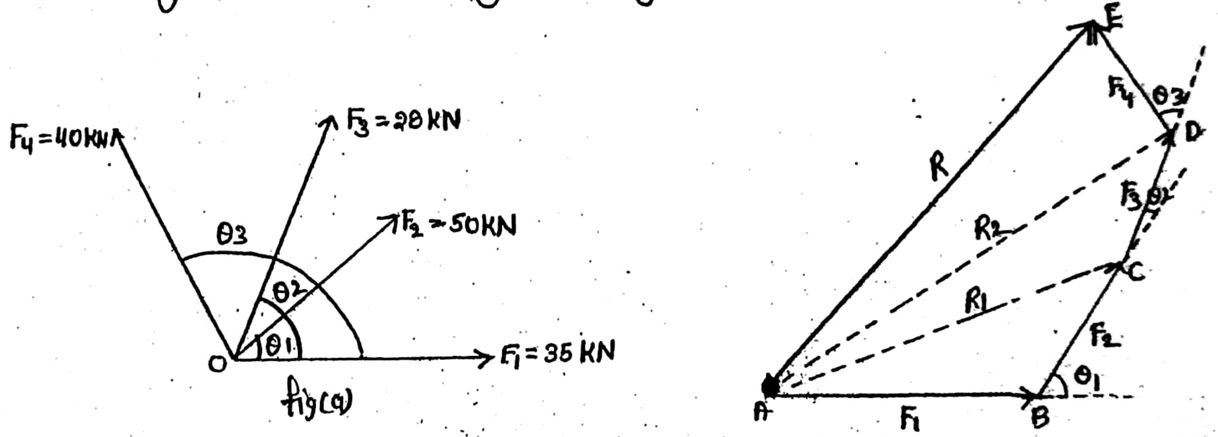


fig (a)

A system of four concurrent forces acting on a body are shown in fig (a). AB represents F_1 and BC represents F_2. Hence according to triangle law of forces AC represents the resultant of F_1 and F_2 say R_1.

If CD is drawn to represents F_3, then from triangle law of force AD represents the resultant of R_1 and F_3. In order AE represents the resultant of F_1, F_2, F_3 and F_4. Thus resultant R is represented by the closing line of the polygon ABCDE in the direction!

GRAPHICAL (VECTOR) METHOD FOR THE RESULTANT FORCE:

It is another name for finding out the magnitude and direction of the resultant force by the polygon law of forces. It is done as discussed below:

1. CONSTRUCTION OF SPACE DIAGRAM (POSITION DIAGRAM):

It means the construction of a diagram showing the various forces (or loads) alongwith their magnitude and lines of action.

2. USE OF BOW'S NOTATIONS:

All the forces in the space diagram are named by using the Bow's notations. It is a convenient method in which every force (or load) is named by two capital letters, placed on its either

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3. CONSTRUCTION OF VECTOR DIAGRAM (FORCE DIAGRAM):

It means the Construction of a diagram starting from a convenient and then go on adding all the forces vectorially one by one (keeping in view the direc of the forces) to some suitable scale.

Now the closing side of the polygon, taken in opposite order, will give the magnitude of the resultant force (to the scale) and its direction.

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MOMENT AND THEIR APPLICATIONS:

MOMENT OF A FORCE:

It is the turning effect produced by a force, on the body, on which it acts. The moment of a force is equal to the product of the force and the perpendicular distance of the point, about which the moment is required and the line of action of the force.

Mathematically, moment

$\boxed{M = P \times L}$ Unit - kN-m, N-mm

Where P = Force acting on the body, and
l = Perpendicular distance between the point, about which the moment is required and the line of action of the force.

GRAPHICAL REPRESENTATION OF MOMENT:

Consider a force P represented, in magnitude and direction, by the line AB. Let O be a point, about which the moment of this force is required to be found out, as shown in fig: in fig. From O, draw OC perpendicular to AB. Join OA and OB.

Now moment of the force P about O.

$= P \times OC = AB \times OC$

But $AB \times OC$ is equal to the twice the area of triangle ABO.

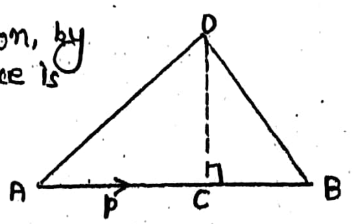


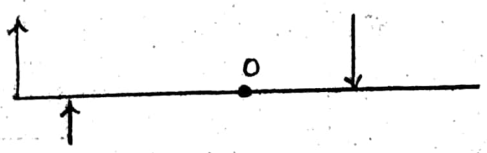
Fig: Representation of Moment

Thus the moment of a force, about any point is equal to twice the area of the triangle, whose base is the line to some scale representing the force and whose vertex is the point about which the moment is taken.

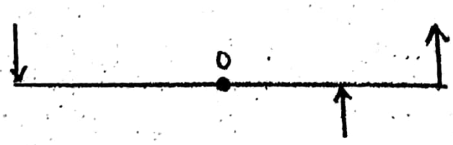
TYPES OF MOMENT:

Broadly speaking the moments are of the following two types:

1. Clockwise moments.
2. Anticlockwise moments.



(a) Clockwise moments



(b) Anticlockwise moments

(a) Clockwise moments

It is the moment of a force, whose effect is to turn or rotate the body, about the point in the same direction in which hands of a clock move as shown in fig: (a).

(b) Anticlockwise moment:

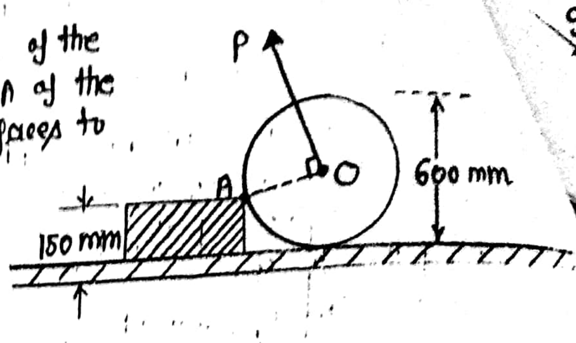
It is the moment of a force, whose effect is to turn or rotate the body, about the point in the opposite direction in which the hands of a clock move as shown in fig (b).

[redacted]

MANISH ARYA

Q: A uniform wheel of 600 mm diameter, weighing 5 KN rests against a rigid block of 150 mm height as shown in fig:

Find the least pull, through the centre of the wheel, required just to turn the wheel over the corner A of the block. Also find the reaction on the block. Take all the surfaces to be smooth.

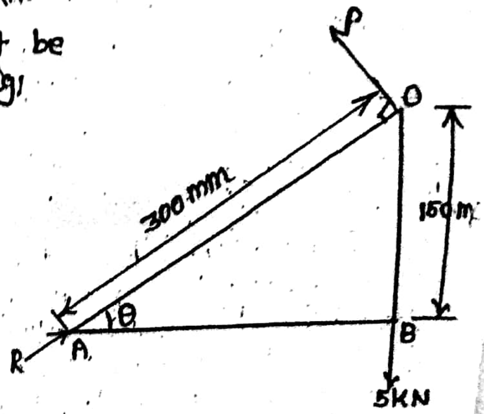


Sol^n: Given; Diameter of wheel = 600 mm;
Weight of wheel = 5 KN;
and height of the block = 150 mm;

Least pull required just to turn the wheel over the corner.

Let = P = Least pull required just to turn the wheel in KN.

A little consideration will show that for the least pull, it must be applied normal to AO. The system of forces is shown in fig.



From the geometry of the fig, we find that,

$\sin\theta = \frac{150}{300} = 0.5$

$\theta = 30°$

and $AB = \sqrt{(300)^2 - (150)^2} = 260$ mm

Now taking moments about A and equating the same,

$P \times 300 = 5 \times 260 = 1300$

$\boxed{P = \frac{1300}{300} = 4.33 \text{ KN}}$ Ans

Reaction on the block:

Let ; R = Reaction on the block in KN.

Resolving the forces horizontally and equating the same.

$R\cos 30° = P\sin 30°$

$\therefore R = \frac{4.33 \times 0.5}{0.866} = 2.5$ KN

$\boxed{R = 2.5 \text{ KN}}$ Ans

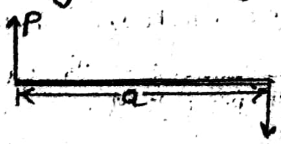
COUPLE:

A Pair of two equal and unlike parallel forces (i.e. forces equal in magnitude, with line of action parallel to each other and acting in opposite directions) is known as a 'couple'.

As a matter of fact, a couple is unable to produce any translatory motion (i.e motion in a straight line). But it produces a motion of rotation in the body, on which it acts. The simple example of a couple is the forces applied to the key of a lock, while locking or unlocking it.

ARM OF A COUPLE:

The perpendicular distance 'a' between the line's of action of the two equal and opposite parallel forces is known as arm of the couple.

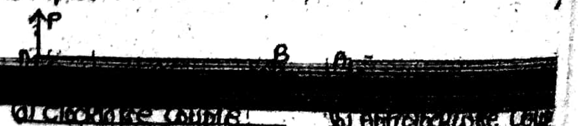


MOMENT OF A COUPLE:

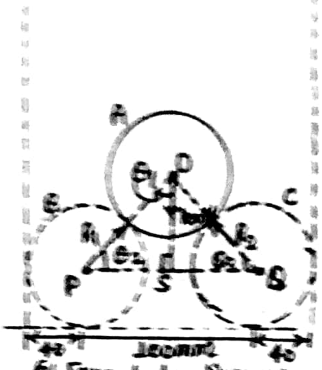
The moment of a couple is the product of the forces (i.e., one of the forces of the two equal and opposite parallel forces), and the arm of the couple.

Mathematically, $\boxed{\text{Moment of a couple} = P \times a}$

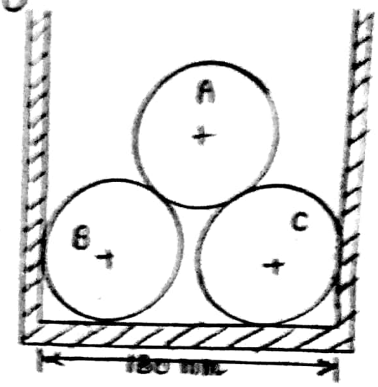
a = Arm of the couple



Que:- Three cylinders weighing 100N each and of 80 mm diameter are placed in a channel of 180 mm width as shown in fig. Determine the pressure exerted by (i) the cylinder A on B at the point of contact (ii) the cylinder B on the base (iii) the cylinder B on the wall.



(i) Free body diagram



Solⁿ From the geometry of the triangle OPS, we find that

$OP = 40 + 40 = 80mm$

$PQ = 100$ mm

$PS = 50$ mm

$\therefore \angle POS = \frac{PS}{OP} = \frac{50}{80} = 0.625$

or $\angle POS = 38.7°$

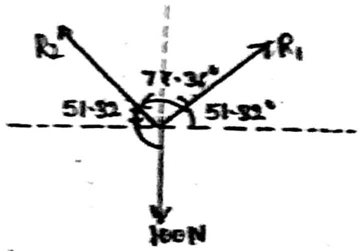
$\theta_1 = 38.7°$

Since the:

$\theta_2 = \theta_3 = 180 - 90 - 38.7$

$\theta_2 = \theta_3 = 51.32$

Since the R_1 and R_2 is acting at an angle 51.32 on cylinder A using lami's theorem.

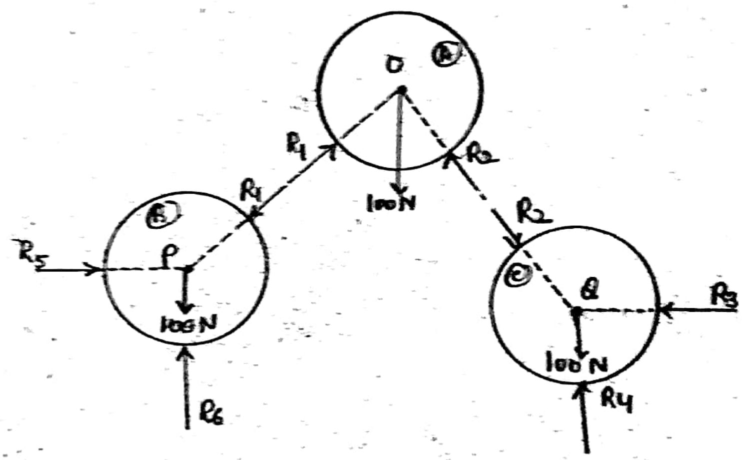


$$\frac{R_1 \text{ (or } R_{AB})}{\sin(90+51.32)} = \frac{100}{\sin 77.36} = \frac{R_2}{\sin(90+51.32)}$$

$$R_{AB} = \frac{100 \times \sin 141.32}{\sin 77.36} = \frac{100 \times 0.6249}{0.9757}$$

$\boxed{R_1 = R_{AB} = 64.06N}$

$\boxed{R_1 = R_2 = 64.06N}$



For cylinder B:

$\Sigma H = 0$

$R_5 = R_1 \cos 51.32$

$R_5 = 64.06 \times \cos 51.32$

$R_5 = 40.03N$

$\Sigma V = 0$

$R_6 = 100 + R_1 \sin\alpha$

$= 100 + 64.06 \cdot \sin 51.32°$

$R_6 = 150N$

For cylinder C:

$\Sigma H = 0$

$R_3 = R_2 \cos\alpha$

$= 64.06 \cos 51.32$

$R_3 = 40.03 N$

$\Sigma V = 0$

$R_4 = R_2 \sin\alpha + 100$

$= 64.06 \times \sin 51.32 + 100$

$R_4 = 150 N$

$R_1 = R_2 = 64.06N$	; $R_3 = R_5 = 40.03N$	$R_4 = R_6 = 150N$

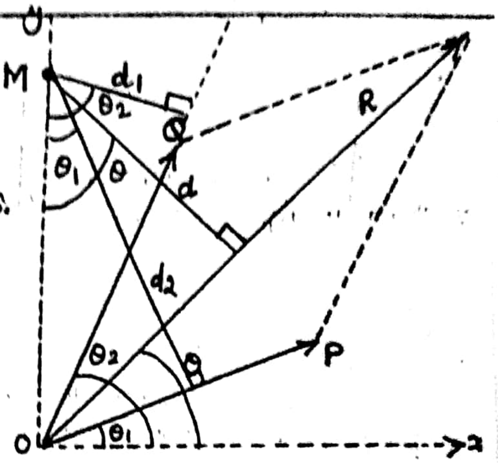
DO

Manish Arya

VARIGNON'S PRINCIPLE OF MOMENTS (OR LAW OF MOMENTS:

It states, " If a number of coplanar forces are acting simultaneously on a particle, the algebraic sum of the moments of all the forces about any point is equal to moment of their resultant force about the same point."

PROOF:



Consider two forces P and Q making angles θ_1 and θ_2 respectively with x-axis and concurrent at the origin as shown in fig. Let their resultant R makes angle θ with x-axis. The perpendicular distances of P, Q and R from point M are d_1, d_2 and d respectively.

The distances d_1, d_2 and d can be expressed in terms of OM as follows:

$$d_1 = OM\cos\theta_2$$
$$d_2 = OM\cos\theta_1$$
$$d = OM\cos\theta$$

The moment of R about point M is:

$$M_R = R \times d = R(OM\cos\theta)$$
$$\therefore M_R = OM\ R\cos\theta$$

The x-component of resultant R is:

$$R_x = R\cos\theta$$
$$\therefore M_R = OM\ R_x \quad \text{........ (1)}$$

The moment of P about M is:

$$M_1 = Pd_2 = P(OM\cos\theta_1)$$
$$\therefore M_1 = OM \cdot P\cos\theta_1$$

But $P\cos\theta_1 = P_x$ is the x-component of P

$$\therefore M_1 = OM \cdot P_x \quad \text{......... (2)}$$

The moment of Q about M is:

$$M_2 = OM\cos\theta_2 \cdot Q = Q \cdot d_1$$
$$= OM\ (Q\cos\theta_2)$$

But $Q\cos\theta_2 = Q_x$ is the x-component of Q

$$M_2 = OM \cdot Q_x \quad \text{...... (3)}$$

Adding equations (1), (ii) and (iii)

$$M_1 + M_2 = OM(P_x + Q_x)$$

But $P_x + Q_x = R_x$ is the x-component of resultant

$$\therefore M_1 + M_2 = OM \cdot R_x$$

from equation (i)

$$\boxed{M_1 + M_2 = M_R}$$

[redacted]

moment of their resultant about M.

LAMI'S THEOREM:

It states, " If three concurrent forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two"

Mathematically,

$$\boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}}$$

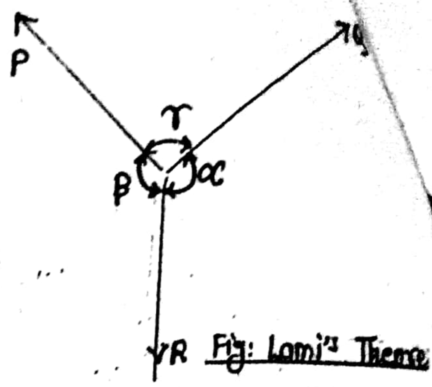


Fig: Lami's Theorem

Where P, Q and R are three forces and α, β, γ are the angles as shown in fig.

PROOF:

Consider three coplanar forces P, Q and R acting at a point O. Let the opposite angles to three forces be α, β and γ as shown in fig.

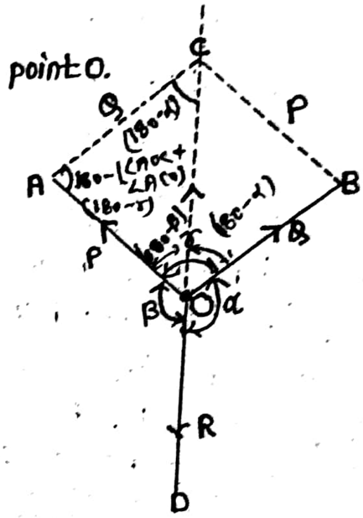


Fig: Proof OF LAMI'S THEOREM

Now let us complete the parallelogram OACB with OA and OB as adjacent sides as shown in fig. we know that the resultant of two forces P and Q will be given by the diagonal OC both in magnitude and direction of the parallelogram OACB.

Since these forces are in equilibrium, therefore the resultant of the forces P and Q must be in line with OD and equal to R, but in opposite direction.

From the geometry of the figure, we find.

BC = P and AC = Q

$\therefore \angle AOC = (180^\circ - \beta)$

and $\angle ACO = \angle BOC = (180^\circ - \alpha)$

$\therefore \angle CAO = 180^\circ - [\angle AOC + \angle ACO]$

$= 180^\circ - [(180 - \beta) + (180^\circ - \alpha)]$

$= 180^\circ - 180^\circ + \beta - 180^\circ + \alpha$

$= \alpha + \beta - 180^\circ$

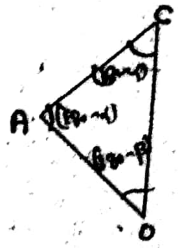
But $\alpha + \beta + \gamma = 360^\circ$

Subtracting 180° from both sides of the above equation,

$(\alpha + \beta - 180^\circ) + \gamma = 360^\circ - 180^\circ = 180^\circ$

or $\angle CAO = 180^\circ - \gamma$

We know that in triangle AOC,



$$\frac{OA}{\sin\angle ACO} = \frac{AC}{\sin\angle AOC} = \frac{OC}{\sin\angle CAO}$$

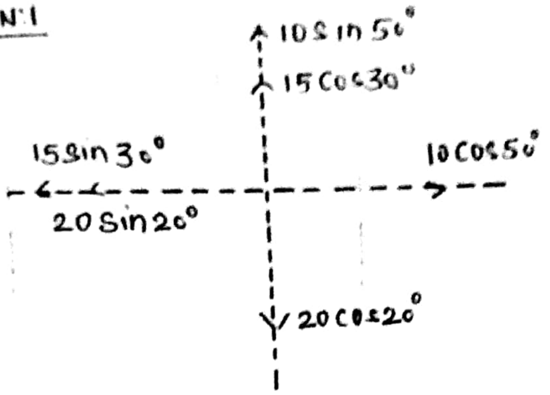
$$\frac{OA}{\sin(180-\alpha)} = \frac{AC}{\sin(180^\circ-\beta)} = \frac{OC}{\sin(180-\gamma)}$$

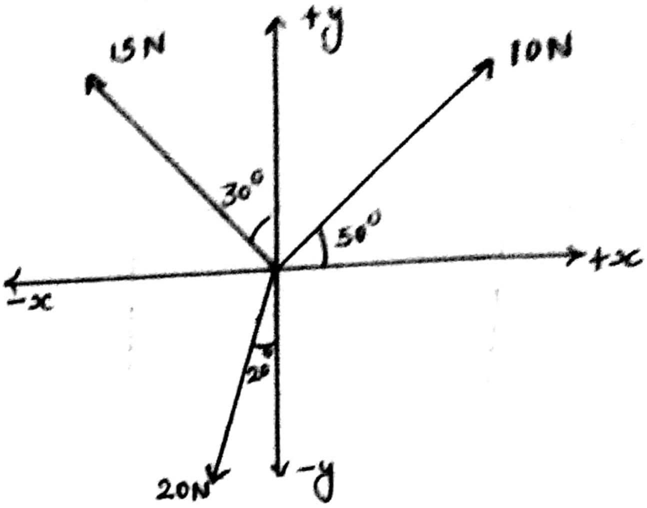
Sinα Sinβ Sinγ

.... $[\sin(180^\circ - \theta) = \sin\theta]$

RESOLUTION OF FORCES

QUESTION: 1



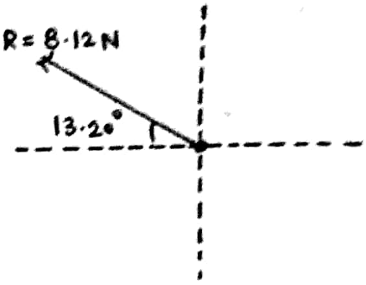


$\Sigma F_x = 0$; $\quad 10\cos 50^\circ - 15\sin 30^\circ - 20\sin 20^\circ = 0$

$\Sigma F_x = -7.912\ N$

$\Sigma F_y = 0$; $\quad 10\sin 50^\circ + 15\cos 30^\circ - 20\cos 20^\circ = 0$

$\Sigma F_y = 1.857\ N$



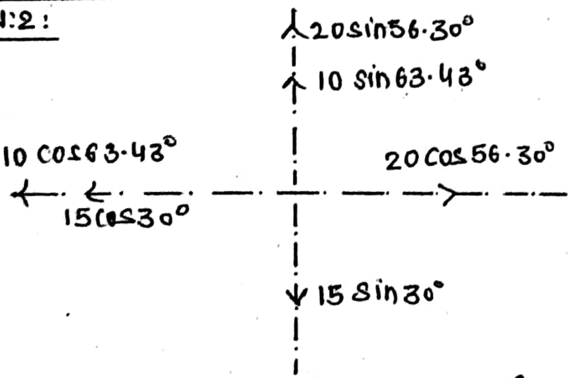
By Applying, $\quad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

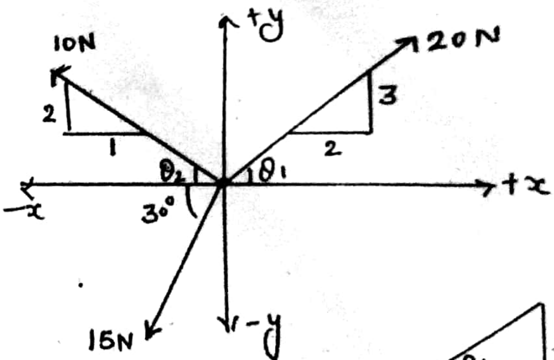
$R = \sqrt{(-7.912)^2 + (1.857)^2}$

$\boxed{R = 8.12\ N}$ $\quad \tan\alpha = \frac{\Sigma F_y}{\Sigma F_x}$, $\quad \alpha = \tan^{-1}\left(\frac{1.857}{7.912}\right) = 13.20^\circ$

$\boxed{\alpha = 13.20^\circ}$

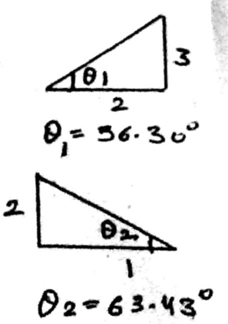
QUESTION: 2:





$\Sigma F_x = 0$; $\quad 20\cos 56.30^\circ - 10\cos 63.43^\circ - 15\cos 30^\circ = 0$

$\Sigma F_x = -6.366\ N$

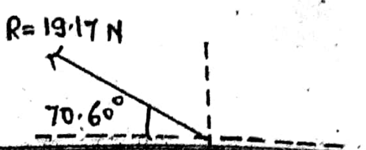


$\Sigma F_y = 0$; $\quad 20\sin 56.30^\circ + 10\sin 63.43^\circ - 15\sin 30^\circ = 0$

$\Sigma F_y = 18.08 N$

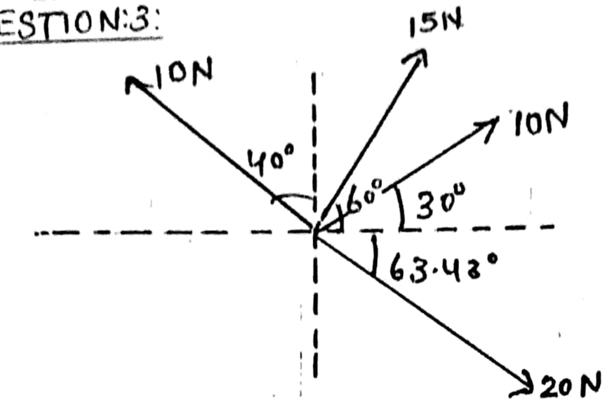
By Applying, $\quad$ Resultant $(R) = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

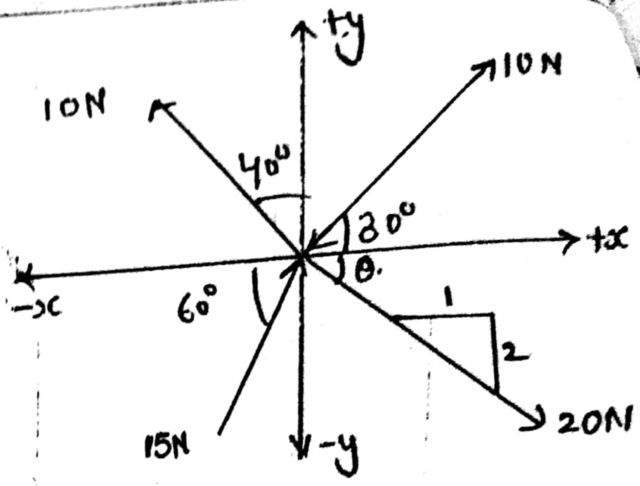
$R = \sqrt{(-6.366)^2 + (18.08)^2}$



$\boxed{R = 19.17\ N}$ $\quad \boxed{\alpha = 70.60^\circ}$

QUESTION:3:



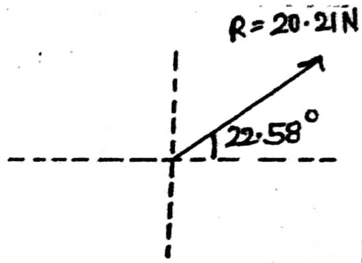


$\Sigma F_x = 0;$ $\quad 10\cos 30° + 15\cos 60° - 10\sin 40° + 20\cos 63.43° = 0$

$$\Sigma F_x = 18.67 N$$

$\Sigma F_y = 0,$ $\quad 10\sin 30° + 15\sin 60° + 10\cos 40° - 20\sin 63.43° = 0$

$$\Sigma F_y = 7.763 N$$



Resultant , $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$$R = \sqrt{(18.67)^2 + (7.763)^2}$$

$$\boxed{R = 20.21 N}$$

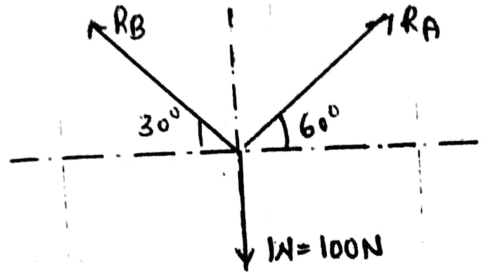
$\tan\alpha = \dfrac{\Sigma F_y}{\Sigma F_x}$; $\alpha = \tan^{-1}\left(\dfrac{7.763 N}{18.67 N}\right)$

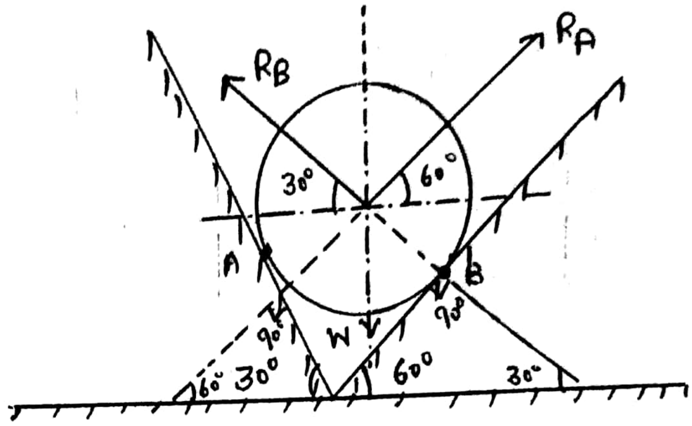
$$\boxed{\alpha = 22.58°}$$

Ans.

SUPPORT REACTIONS:

QUESTION :1: $W = 100N$





By applying Lami's Theorem:

$$\frac{100}{\sin 90^\circ} = \frac{R_A}{\sin 120^\circ} = \frac{R_B}{\sin 150^\circ}$$

$$R_A = 100 \times \frac{\sin 120^\circ}{\sin 90^\circ} = \boxed{R_A = 86.60N}$$

$$R_B = 100 \times \frac{\sin 150^\circ}{\sin 90^\circ} = \boxed{R_B = 50.0N}$$

By applying resolution of forces:

$\Sigma F_x = 0$: $\quad R_A \cos 60^\circ = R_B \cos 30^\circ$

$$R_A = 1.732\, R_B \quad \ldots (i)$$

$\Sigma F_y = 0$: $\quad R_A \sin 60^\circ + R_B \sin 30^\circ = 100$

$$(1.732\, R_B) \sin 60^\circ + R_B \sin 30^\circ = 100$$

$$R_B (1.732 \times \sin 60^\circ + \sin 30^\circ) = 100$$

$$R_B = 50N$$

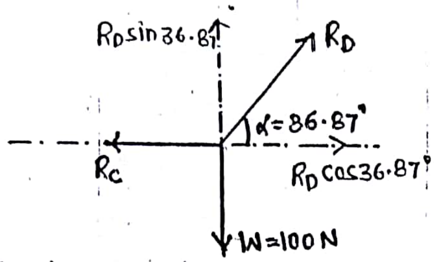
$$R_A = 1.732\, R_B = 1.732 \times 50 = 86.60N.$$

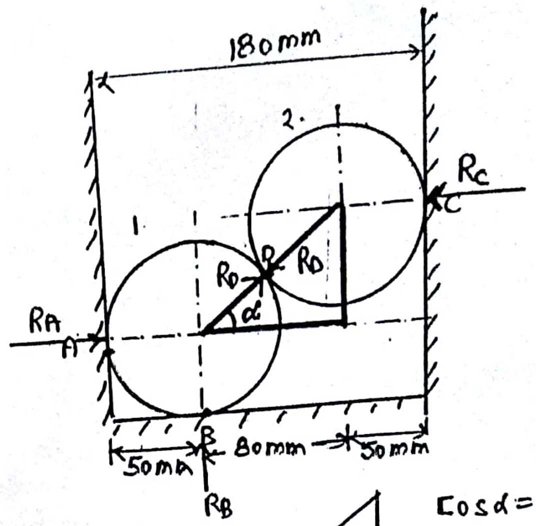
$\boxed{R_A = 86.60N}$, $\boxed{R_B = 50.0N}$

QUESTION:2: $W = 100N$, $R = 50mm$

Solution:2:

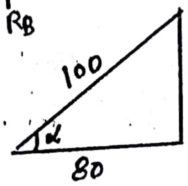
F.B.D of sphere:2:





By Applying Lami's Theorem:

$$\frac{100}{\sin 143.13} = \frac{R_C}{\sin 126.87°} = \frac{R_D}{\sin 90°}$$



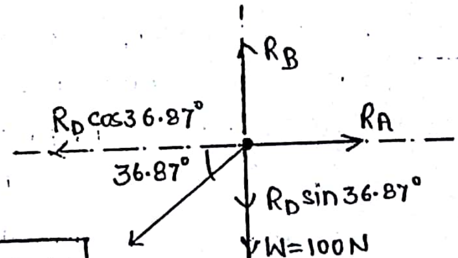
$\cos\alpha = \frac{80}{100}$

$\alpha = 36.87°$

$$R_C = 100 \times \frac{\sin 126.87°}{\sin 143.13} = 133.33\ N$$

$$R_D = 100 \times \frac{\sin 90°}{\sin 143.13°} = 166.67\ N$$

F.B:D of sphere:1



By Applying equilibrium equations:

$\Sigma F_x = 0;$ $R_A = 166.67 \cos 36.87°$

$R_A = 133.33N$

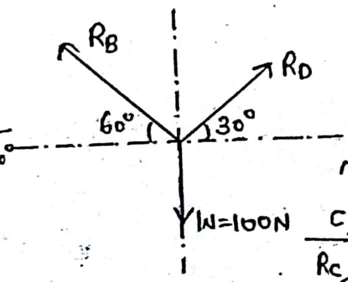
$\Sigma F_y = 0;$ $R_B = 100 + 166.67 \sin 36.87°$

$R_B = 200N$

$\boxed{R_A = 133.33N}$ $\boxed{R_B = 200N}$ $\boxed{R_C = 133.33N}$ $\boxed{R_D = 166.67N}$

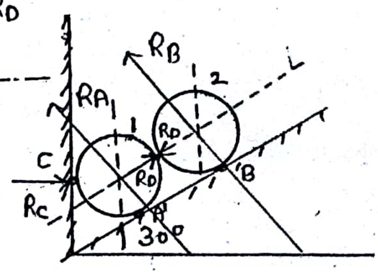
QUESTION:3: $W = 100N$, $R = 50mm$.

Solution: F.B.D of sphere:2:



By applying Lami's Theorem:

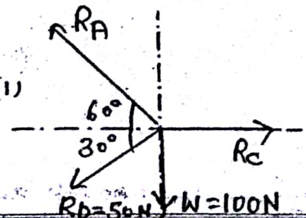
$$\frac{100}{\sin 90°} = \frac{R_B}{\sin 120°} = \frac{R_D}{\sin 150°}$$



$$R_B = 100 \times \frac{\sin 120°}{\sin 90°} = 86.66N$$

$$R_D = 100 \times \frac{\sin 150°}{\sin 90°} = 50.0N$$

F.B.D of sphere:1:



$\Sigma F_x = 0;$ $R_C = R_A \cos 60° + 50 \cos 30°$ ...(1)

$\Sigma F_y = 0;$ $R_A \sin 60° = 100 + 50 \sin 30°$

$R_A = 144.33N$

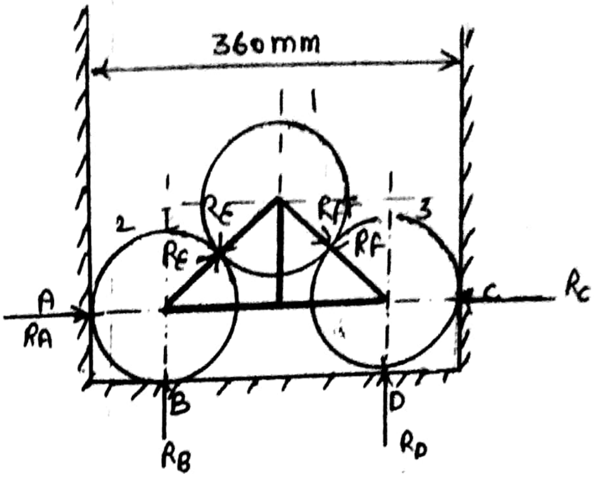
$\boxed{R_A = 144.33N}$

$\boxed{R_B = 86.60N}$

$\boxed{R_C = 115.46N}$

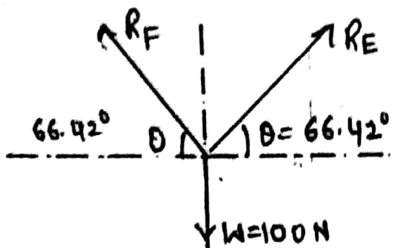
$\boxed{R_D = 50.0N}$

QUESTION:4: $W = 100\,N$, $R = 100\,mm$



Solution: 4:

F.B.D of sphere:1

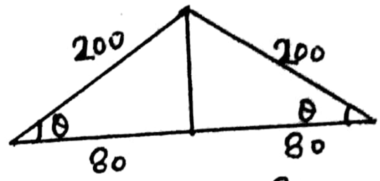


By applying Lami's Theorem:

$$\frac{100}{\sin 47.16} = \frac{R_E}{\sin 156.42^\circ} = \frac{R_F}{\sin 156.42^\circ}$$

$$R_E = 100 \times \frac{\sin 156.42}{\sin 47.16} = 54.55\,N$$

$$R_F = 100 \times \frac{\sin 156.42}{\sin 47.16} = 54.55\,N$$



$$\cos\theta = \frac{80}{200}$$

$$\theta = 66.42^\circ$$

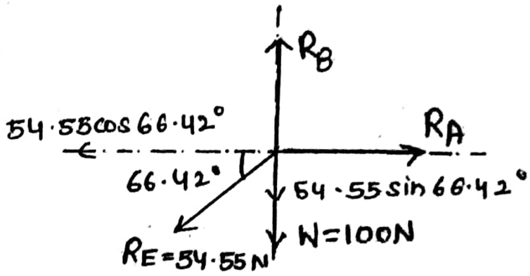
F.B.D of sphere:2:

$\Sigma F_x = 0$; $R_A = 54.55 \cos 66.42^\circ$

$R_A = 21.82\,N$

$\Sigma F_y = 0$; $R_B = 100 + 54.55 \sin 66.42^\circ$

$R_B = 100.91\,N$



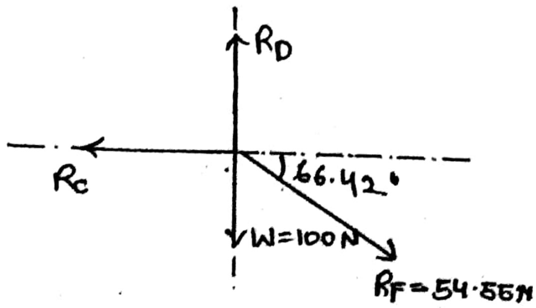
F.B.D of sphere:3:

$\Sigma F_x = 0$; $R_C = 54.55 \cos 66.42^\circ$

$R_C = 21.82\,N$

$\Sigma F_y = 0$; $R_D = 100 + 54.55 \sin 66.42^\circ$

$R_D = 100.91\,N$



$\boxed{R_A = 21.82\,N}$

$\boxed{R_B = 100.91\,N}$

$\boxed{R_C = 21.82\,N}$

$\boxed{R_D = 100.91\,N}$

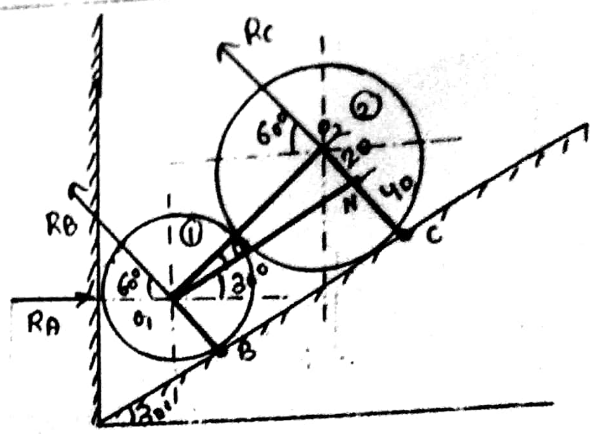
$\boxed{R_E = 54.55\,N}$

Ans.

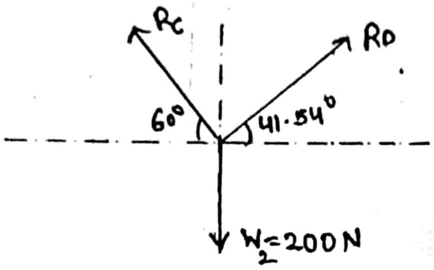
QUESTION: 5: $R_1 = 40mm$, $R_2 = 60mm$

$W_1 = 100N$, $W_2 = 200N$

ΔO_1O_2N: $\sin\theta = \frac{20}{100}$, $\theta = 11.53°$



F.B.D of sphere (2):



By applying Lami's Theorem:

$$\frac{200}{\sin(180-60-41.54°)} = \frac{R_C}{\sin(131.54°)} = \frac{R_D}{\sin 150°}$$

$$R_C = 200 \times \frac{\sin 131.54°}{\sin 78.46°} = 152.787\ N$$

$$R_D = 200 \times \frac{\sin 150°}{\sin 78.46°} = 102.063\ N$$

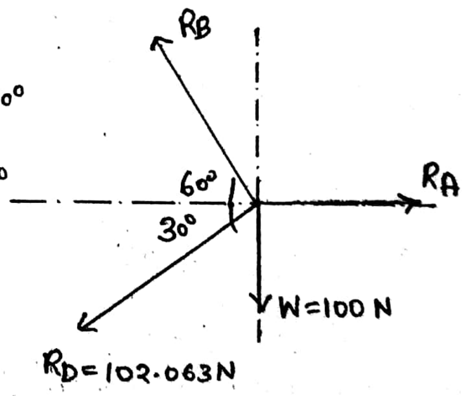
F.B.D of sphere: 1:

$\Sigma F_x = 0$; $R_A = R_B \cos 60° + 102.063 \cos 30°$

$\Sigma F_y = 0$; $R_B \sin 60° = 100 + 102.063 \sin 30°$

$R_B = 174.396\ N$

$R_A = 175.587\ N$



$\boxed{R_A = 175.587\ N}$

$\boxed{R_B = 174.396\ N}$

$\boxed{R_C = 152.787\ N}$

$\boxed{R_D = 102.063\ N}$

Ans